

Math 1170  
Mock Exam

1 [5] Let  $f(x) = \frac{1}{x+1}$ , calculate and simplify  $\frac{f(x+h) - f(x)}{h}$ , where  $h$  is a small positive constant.

2 A restaurant charges \$10 per person for a buffet. The average number of customer is 120 per day. From previous experience, the owner estimates that for every \$.50 increases in price the number of customer decreases by 4 per day.

a [3] Let  $x$  be the price per person. Express  $R$  = the revenue per day as a function of  $x$ .

b [2] If the restaurant pays an overhead of \$100 per day, and the food cost is \$4 per customer, express  $P$  = profit per day as a function of  $x$ .

c [2] Based on your answer in (b), what price should the restaurant set to maximize its profit?

3 Find all the real solutions to each of the following equation.

a [4]  $\sqrt{x+1} + \sqrt{x-2} = 2$     b [4]  $1 - \sin(t) = \sqrt{3} \cos(t)$      $t \in [0, 2\pi]$

c [4]  $e^x + e^{-x} = 1$     d [4]  $\cos(t) - \sin(2t) = 0$ ,     $t \in [0, 2\pi]$

e  $\arcsin \sqrt{x} = \frac{\pi}{6}$

4 Let  $y = f(x) = \frac{3x^3 - 4x^2 + x}{2x^2 + 5x - 3}$

a [6] Sketch  $f(x)$ , show all asymptotes and zeros.

b [2] Does the inverse of  $f(x)$  exist? Why or why not?

5 Let  $f(x) = \frac{1}{1+e^x}$

a [2] Find  $f'(x)$

b [2] What is the domain of  $f'(x)$ ?

c [4] Sketch  $f'(x)$  Indicate all asymptotes and zeros.

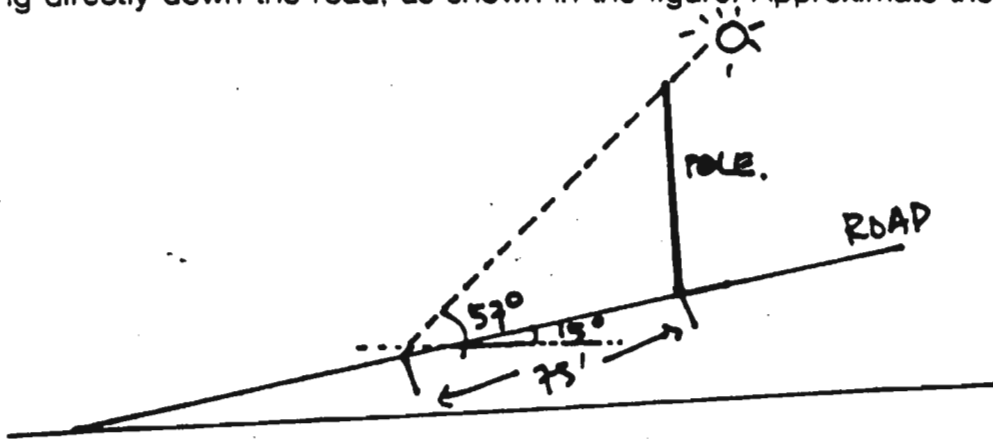
6 A drug is eliminated from the body through the kidney in such a way that over each hour, 25% of the amount present at the beginning of the hour is eliminated. Let  $t$  = the amount of time (in hours) since the drug was first taken, and  $A_0$  = the initial amount of the drug in the body (at time  $t=0$ ).

- a [4] Write down the expression of  $A(t)$  = the amount of drug in the body  $t$  hours after the drug is taken, in terms of  $A_0$  and  $t$ .
- b [2] How long does it take before the amount of drug in the body is half of the initial amount? (ie, what is the half life of the drug?).

7 The body temperature varies according to a rhythm that repeats itself every 24 hours. Body temperature is highest around 5 pm, and is lowest around 5 am. Let  $T(t)$  = the body temperature (in °F) at time  $t$ , where  $t=0$  correspond to midnight. The low and high body temperatures are 98.3°F and 98.9°F.

- a [4] Find an equation of the form  $T(t) = 98.6 + A \sin(bt+c)$  that fits this information. ( $b > 0$ )
- b [4] For what percentage of the time does the body temperature exceed 98.8°F?

8 [5] (p.359 # 20) A straight road makes an angle of 15% with the horizontal. When the angle of elevation of the sun is 57°, a vertical pole at the side of the road cast a shadow 75 feet long directly down the road, as shown in the figure. Approximate the length of the pole.



9 [4] (p. 326 # 48) If a particle is fired from a ground level with an initial velocity of  $v$  ft/sec and at an angle of  $\theta$  with the horizontal, the range  $R$  of the projectile is given by:

$$R = \frac{v^2}{16} \sin(\theta) \cos(\theta)$$

If  $v = 80$  ft/sec. approximate the angles that result in a range of 150 ft.

10 Verify or disprove the following:

a[4]  $\sin(3u) = \sin u(3 - 4\sin^2(u))$

b[4]  $-\ln|\sec(\theta) - \tan(\theta)| = \ln|\sec(\theta) + \tan(\theta)|$

c[4]  $\cos\left(\theta + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos\theta - \sin\theta)$

d[4]  $\csc\alpha = \sin\left(\frac{1}{\alpha}\right)$

e  $\arctan x = \frac{\arcsin x}{\arccos x}$

Note: The above represents about the length, difficulty level, and topic distribution of a typical Math 1170 exam, but of course the individual question types might not be exactly the same. In particular, with graphing calculators permitted, questions like #4 and 5 above would have graphs of  $y=f(x)$  provided, and the marks would be given for the exact locations and equations of intercepts and asymptotes. Examples of a couple of other question types that might be included are given on the following page.

- I a) Sketch a graph of a function  $f$  that has all of the following properties.
- (i)  $f$  has  $x$ -intercepts at 0, 1 and -2.
  - (ii) as  $x \rightarrow -1$ ,  $f(x) \rightarrow \infty$
  - (iii)  $y = x - 1$  is an asymptote for  $f$

b) Find a possible formula for the above function

II The graph of a function  $f$  is shown here.

On the axes below sketch graphs of

a)  $g \circ f$

b)  $f \circ g$

where  $g(x) = |x|$  is the absolute value function.

