

## Answers to Problem Set 10

Total 50 marks

1. [6 marks]

1. $\forall x \text{ Cube}(x)$	
2. $\forall y \text{ Small}(y)$	
3. $c$	
4. $\text{Cube}(c)$	✓ $\forall \text{ Elim} :1$
5. $\text{Small}(c)$	✓ $\forall \text{ Elim} :2$
6. $\text{Small}(c) \wedge \text{Cube}(c)$	✓ $\wedge \text{ Intro} :4,5$
7. $\forall z (\text{Small}(z) \wedge \text{Cube}(z))$	✓ $\forall \text{ Intro} :3-6$

2. [6 marks]

1. $\exists y \text{ Cube}(y) \vee \exists z \text{ Small}(z)$	
2. $\exists y \text{ Cube}(y)$	
3. $a$ $\text{Cube}(a)$	
4. $\text{Cube}(a) \vee \text{Small}(a)$	✓ $\vee \text{ Intro} :3$
5. $\exists x (\text{Cube}(x) \vee \text{Small}(x))$	✓ $\exists \text{ Intro} :4$
6. $\exists x (\text{Cube}(x) \vee \text{Small}(x))$	✓ $\exists \text{ Elim} :2,3-5$
7. $\exists z \text{ Small}(z)$	
8. $b$ $\text{Small}(b)$	
9. $\text{Cube}(b) \vee \text{Small}(b)$	✓ $\vee \text{ Intro} :8$
10. $\exists x (\text{Cube}(x) \vee \text{Small}(x))$	✓ $\exists \text{ Intro} :9$
11. $\exists x (\text{Cube}(x) \vee \text{Small}(x))$	✓ $\exists \text{ Elim} :7,8-10$
12. $\exists x (\text{Cube}(x) \vee \text{Small}(x))$	✓ $\vee \text{ Elim} :1,7-11,2-6$

3. [6 marks]

1. $\exists x \exists y (Cube(x) \wedge Cube(y) \wedge \neg SameRow(x,y))$	
2. $\boxed{a \ b} Cube(a) \wedge Cube(b) \wedge \neg SameRow(a,b)$	
3. $\forall x \forall y ((Cube(x) \wedge Cube(y)) \rightarrow SameRow(x,y))$	
4. $(Cube(a) \wedge Cube(b)) \rightarrow SameRow(a,b)$	✓ $\forall$ Elim :3
5. $Cube(a) \wedge Cube(b)$	✓ $\wedge$ Elim :2
6. $SameRow(a,b)$	✓ $\rightarrow$ Elim :4,5
7. $\neg SameRow(a,b)$	✓ $\wedge$ Elim :2
8. $\perp$	✓ $\perp$ Intro :6,7
9. $\neg \forall x \forall y ((Cube(x) \wedge Cube(y)) \rightarrow SameRow(x,y))$	✓ $\neg$ Intro :3-8
10. $\neg \forall x \forall y ((Cube(x) \wedge Cube(y)) \rightarrow SameRow(x,y))$	✓ $\exists$ Elim :2-9,1

4. [6 marks]

1. $\neg \exists x Cube(x)$	
2. $\boxed{a}$	
3. $Cube(a)$	
4. $\exists x Cube(x)$	✓ $\exists$ Intro :3
5. $\perp$	✓ $\perp$ Intro :1,4
6. $\neg Cube(a)$	✓ $\neg$ Intro :3-5
7. $\forall x \neg Cube(x)$	✓ $\forall$ Intro :2-6

5. (a) Write the argument as it appears through FO goggles [1 mark]

Original argument:

$$\frac{\exists x(\text{SameShape}(x, a) \wedge \text{Cube}(x))}{\neg(\text{Tet}(a) \vee \text{Dodec}(a))}$$

FO goggles:

$$\frac{\exists x(P(x, a) \wedge Q(x))}{\neg(R(a) \vee S(a))}$$

- (b) My interpretation of the nonsense predicates used in part (a): [2 marks]

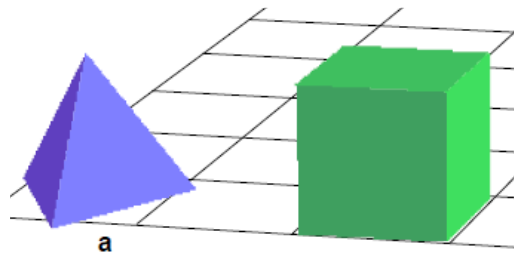
Turn  $P(x, y)$  into  $\text{SameSize}(x, y)$   
 Turn  $Q, R, S$  back into  $\text{Cube}, \text{Tet}, \text{Dodec}$ .

(SameRow, SameCol, LeftOf, Larger, Loves, etc. would also work. Or change  $Q(x)$  into  $\text{Small}(x)$ , etc.)

- (c) Draw a counter-example world, for the new argument (with the new predicates)

$$\frac{\text{T} \quad \exists x(\text{SameSize}(x, a) \wedge \text{Cube}(x))}{\text{F} \quad \neg(\text{Tet}(a) \vee \text{Dodec}(a))}$$

[2 marks]



6. [7 marks]

N.B. I listed the 3 axioms used as premises, so I could use them in Fitch. But this isn't necessary for students to do.

1.  $\exists x (\text{SameShape}(x, a) \wedge \text{Cube}(x))$
2.  $\forall x \forall y ((\text{SameShape}(x, y) \wedge \text{Cube}(x)) \rightarrow \text{Cube}(y))$  (A8)
3.  $\neg \exists x (\text{Cube}(x) \wedge \text{Tet}(x))$  (A1)
4.  $\neg \exists x (\text{Dodec}(x) \wedge \text{Cube}(x))$  (A3)
5.  $\nabla \text{Tet}(a) \vee \text{Dodec}(a)$
6.  $\boxed{b} \nabla \text{SameShape}(b, a) \wedge \text{Cube}(b)$
7.  $(\text{SameShape}(b, a) \wedge \text{Cube}(b)) \rightarrow \text{Cube}(a)$  ✓  $\nabla \forall$  Elim: A8
8.  $\text{Cube}(a)$  ✓  $\nabla \rightarrow$  Elim: 6,7
9.  $\nabla \text{Tet}(a)$
10.  $\text{Cube}(a) \wedge \text{Tet}(a)$  ✓  $\nabla \wedge$  Intro: 9,8
11.  $\exists x (\text{Cube}(x) \wedge \text{Tet}(x))$  ✓  $\nabla \exists$  Intro: 10
12.  $\perp$  ✓  $\nabla \perp$  Intro: 11, A1
13.  $\nabla \text{Dodec}(a)$
14.  $\text{Dodec}(a) \wedge \text{Cube}(a)$  ✓  $\nabla \wedge$  Intro: 13,8
15.  $\exists x (\text{Dodec}(x) \wedge \text{Cube}(x))$  ✓  $\nabla \exists$  Intro: 14
16.  $\perp$  ✓  $\nabla \perp$  Intro: 15, A3
17.  $\perp$  ✓  $\nabla \vee$  Elim: 13-16, 9-12, 5
18.  $\perp$  ✓  $\nabla \exists$  Elim: 6-17, 1
19.  $\neg(\text{Tet}(a) \vee \text{Dodec}(a))$  ✓  $\nabla \neg$  Intro: 5-18

7. [7 marks]

- |  |                           |
|--|---------------------------|
| 1. $\exists x (\text{Dog}(x) \wedge \text{Lazy}(x))$   |                           |
| 2. $\forall x \forall y ((\text{Dog}(x) \wedge \text{Dog}(y)) \rightarrow x = y)$                        |                           |
| <hr/>  |                           |
| 3. $\boxed{a} \text{Dog}(a) \wedge \text{Lazy}(a)$   |                           |
| 4. $\text{Dog}(a)$   | ✓ $\wedge$ Elim :3        |
| 5. $\text{Lazy}(a)$  | ✓ $\wedge$ Elim :3        |
| <hr/>  |                           |
| 6. $\boxed{b} \text{Dog}(b)$   |                           |
| 7. $\text{Dog}(a) \wedge \text{Dog}(b)$  | ✓ $\wedge$ Intro :4,6     |
| 8. $(\text{Dog}(a) \wedge \text{Dog}(b)) \rightarrow a = b$  | ✓ $\forall$ Elim :2       |
| 9. $a = b$   | ✓ $\rightarrow$ Elim :7,8 |
| 10. $\forall y (\text{Dog}(y) \rightarrow a = y)$  | ✓ $\forall$ Intro :6-9    |
| 11. $\text{Dog}(a) \wedge \forall y (\text{Dog}(y) \rightarrow a = y) \wedge \text{Lazy}(a)$             | ✓ $\wedge$ Intro :4,5,10  |
| 12. $\exists x (\text{Dog}(x) \wedge \forall y (\text{Dog}(y) \rightarrow x = y) \wedge \text{Lazy}(x))$ | ✓ $\exists$ Intro :11     |
| 13. $\exists x (\text{Dog}(x) \wedge \forall y (\text{Dog}(y) \rightarrow x = y) \wedge \text{Lazy}(x))$ | ✓ $\exists$ Elim :3-12,1  |

8. [7 marks]

- |  |                                  |
|--|----------------------------------|
| 1. $\forall x (\exists y \text{Loves}(x, y) \rightarrow \text{Loves}(x, \text{celine}))$             |                                  |
| 2. $\text{Loves}(\text{celine}, \text{bill})$  |                                  |
| 3. $\neg \text{Loves}(\text{alice}, \text{alice})$   |                                  |
| <hr/>  |                                  |
| 4. $\nabla \text{alice} = \text{celine}$   |                                  |
| 5. $\exists y \text{Loves}(\text{celine}, y) \rightarrow \text{Loves}(\text{celine}, \text{celine})$ | ✓ $\nabla \forall$ Elim: 1       |
| 6. $\exists y \text{Loves}(\text{celine}, y)$  | ✓ $\nabla \exists$ Intro: 2      |
| 7. $\text{Loves}(\text{celine}, \text{celine})$  | ✓ $\nabla \rightarrow$ Elim: 5,6 |
| 8. $\neg \text{Loves}(\text{celine}, \text{celine})$   | ✓ $\nabla =$ Elim: 3,4           |
| 9. $\perp$   | ✓ $\nabla \perp$ Intro: 7,8      |
| 10. $\text{alice} \neq \text{celine}$  | ✓ $\nabla \neg$ Intro: 4-9       |