

Basic translations

All cubes are small.	$\forall x(\text{Cube}(x) \rightarrow \text{Small}(x))$		
Some cubes are small.	$\exists x(\text{Cube}(x) \wedge \text{Small}(x))$		
No cube is small.	$\forall x(\text{Cube}(x) \rightarrow \neg \text{Small}(x))$	OR	$\neg \exists x(\text{Cube}(x) \wedge \text{Small}(x))$
Some cubes are not small.	$\exists x(\text{Cube}(x) \wedge \neg \text{Small}(x))$	OR	$\neg \forall x(\text{Cube}(x) \rightarrow \text{Small}(x))$
Only cubes are small.	$\forall x(\text{Small}(x) \rightarrow \text{Cube}(x))$		

At Least

There is at least 1 cube.	$\exists x \text{Cube}(x)$
There are at least 2 cubes.	$\exists x \exists y (\text{Cube}(x) \wedge \text{Cube}(y) \wedge x \neq y)$
There are at least 3 cubes.	$\exists x \exists y \exists z (\text{Cube}(x) \wedge \text{Cube}(y) \wedge \text{Cube}(z) \wedge x \neq y \wedge y \neq z \wedge z \neq x)$

Exactly

There is exactly 1 cube.	$\exists x(\text{Cube}(x) \wedge \forall y(\text{Cube}(y) \rightarrow y=x))$
There are exactly 2 cubes.	$\exists x \exists y (\text{Cube}(x) \wedge \text{Cube}(y) \wedge x \neq y \wedge \forall z(\text{Cube}(z) \rightarrow (z=x \vee z=y)))$
There are exactly 3 cubes.	$\exists x \exists y \exists z (\text{Cube}(x) \wedge \text{Cube}(y) \wedge \text{Cube}(z) \wedge x \neq y \wedge y \neq z \wedge z \neq x \wedge \forall w(\text{Cube}(w) \rightarrow (w=x \vee w=y \vee w=z)))$

At Most

There is at most 1 cube.	$\forall x \forall y ((\text{Cube}(x) \wedge \text{Cube}(y)) \rightarrow x=y)$
There are at most 2 cubes.	$\forall x \forall y \forall z ((\text{Cube}(x) \wedge \text{Cube}(y) \wedge \text{Cube}(z)) \rightarrow (x=y \vee y=z \vee z=x))$
There are at most 3 cubes.	$\forall x \forall y \forall z \forall w ((\text{Cube}(x) \wedge \text{Cube}(y) \wedge \text{Cube}(z) \wedge \text{Cube}(w)) \rightarrow (x=y \vee x=z \vee x=w \vee y=z \vee y=w \vee z=w))$

'The'

The cube is small.	$\exists x(\text{Cube}(x) \wedge \forall y(\text{Cube}(y) \rightarrow y=x) \wedge \text{Small}(x))$
The smallest object is c .	$\forall x(x \neq c \rightarrow \text{Smaller}(c,x))$
The smallest object is a cube.	$\exists x(\text{Cube}(x) \wedge \forall y(y \neq x \rightarrow \text{Smaller}(x,y)))$

Donkey

Every cube adjoins a tet is also smaller than it.

Standard Donkey (Stronger) $\forall x \forall y ((\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{Adjoins}(x, y)) \rightarrow \text{Smaller}(x, y))$

Donkey-coop (Weaker) $\exists y (\text{Tet}(y) \wedge \forall x ((\text{Cube}(x) \wedge \text{Adjoins}(x, y)) \rightarrow \text{Smaller}(x, y)))$

Mother / Bieber

Every cube adjoins a tet.

Bieber (Stronger) $\exists y (\text{Tet}(y) \wedge \forall x (\text{Cube}(x) \rightarrow \text{Adjoins}(x, y)))$

Mother (Weaker) $\forall x (\text{Cube}(x) \rightarrow \exists y (\text{Tet}(y) \wedge \text{Adjoins}(x, y)))$

Not Every

a is not smaller than every cube.

Negated Existential (Stronger) $\neg \exists x (\text{Cube}(x) \wedge \text{Smaller}(a, x))$

Negated Universal (Weaker) $\neg \forall x (\text{Cube}(x) \rightarrow \text{Smaller}(a, x))$

No Some

No cube adjoins some tet.

Negated Existential (Stronger) $\neg \exists x \exists y (\text{Cube}(x) \wedge \text{Tet}(y) \wedge \text{Adjoins}(x, y))$

Existential Negation (Weaker) $\exists y (\text{Tet}(y) \wedge \neg \exists x (\text{Cube}(x) \wedge \text{Adjoins}(x, y)))$

Emergency Tier

Tier	Rules	Subproof needed?
0 (Very emergency)	\wedge Elim, \rightarrow Elim, \leftrightarrow Elim, M.T., D.S.	No
1	\neg Intro, \rightarrow Intro, \leftrightarrow Intro, \forall Intro	Yes (Fresh for \forall Intro)
2	\exists Elim	Yes (Fresh)
3	\forall Elim	No
4	\forall Elim	Yes
5 (Not so emergency)	All the others	No