Philosophy 1102
Introduction to Logic

Answers to Problem Set 9

Total: 50 marks

Note: In these translations there are many possible answers! *Any answer that's FO equivalent to mine is also fully correct.*

1. [6 marks total]

	Partial translations	**missing part to add in FOL**
(i)	∀x ((Cube(x) ∧ Small(x)) → ∃y (Large(y) ∧ Cube(y) ∧ BackOf(x, y)))	x-is-in-back-of-a-large-cube
(ii)	∃x (Cube(x) ∧ ∀y (Tet(y) → FrontOf(x, y)))	x-is-in-front-of-every-tet
(iii)	∃x ((Cube(x) ∧ Large(x)) ∧ ∃y (Small(y) ∧ Cube(y) ∧ FrontOf(x, y)))	x-is-in-front-of-a-small-cube
(iv)	∀x (∃y (Large(y) ∧ Cube(y) ∧ RightOf(x, y)) → Small(x))	x-is-to-the-right-of-a-large-cube
(v)	∀x(∀y ¬ BackOf(y, x) → Cube(x))	there-is-nothing-in-back-of-x
(vi)	∀x (Dodec(x) → ∃y (Tet(y) ∧ Smaller(x, y)))	x-is-smaller-than-some-tet

(i) Every small cube is in back of a large cube.
(ii) Some cube is in front of every tetrahedron.
(iii) A large cube is in front of a small cube.
(iv) Everything to the right of a large cube is small.
(v) Anything with nothing in back of it is a cube.
(vi) Every dodecahedron is smaller than some tetrahedron.

2. Translate the following sentences into FOL. If correct, your answers will all be true in the world given below, as well as in Ron's World. [2 marks each, total 10]

(i) Some dodecahedron is such that all the tetrahedra in its row are medium.
(ii) If two tetrahedra are in the same row, then they're the same size.
(iii) Every tetrahedron that adjoins a cube is the same size as it.
(iv) Only large objects have nothing in front of them.
(v) If e is between two objects, then they (the two objects) are both small.

(i) $\exists x\,(Dodec(x) \wedge \forall y\,((Tet(y) \wedge SameRow(x, y)) \rightarrow Medium(y)))$

(ii) $\forall x \forall y\,((Tet(x) \wedge Tet(y) \wedge SameRow(x, y)) \rightarrow SameSize(x,y))$

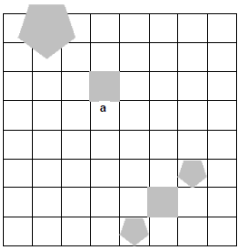
(iii) $\forall x \forall y\,((Tet(x) \wedge Cube(y) \wedge Adjoins(x, y)) \rightarrow SameSize(x, y))$

(iv) $\forall x\,(\neg \exists y\, FrontOf(y, x) \rightarrow Large(x))$

(v) $\forall x \forall y\,(Between(e, x, y) \rightarrow (Small(x) \wedge Small(y)))$

3. [2 marks for each FOL sentence, total 16]

(i) Every cube is between a pair of dodecahedra.
(ii) Every cube to the right of a dodecahedron is smaller than it is.
(iii) a is not larger than every dodecahedron.
(iv) No cube is to the left of some dodecahedron.



(i) a. $\forall x\,(Cube(x) \rightarrow \exists y \exists z\,(Dodec(y) \wedge Dodec(z) \wedge Between(x, y, z))$
b. $\exists y \exists z\,(Dodec(y) \wedge Dodec(z) \wedge \forall x(Cube(x) \rightarrow Between(x, y, z)))$

(ii) a. $\exists y\,(Dodec(y) \wedge \forall x\,((Cube(x) \wedge RightOf(x, y)) \rightarrow Smaller(x, y))$
b. $\forall x \forall y\,((Cube(x) \wedge Dodec(y) \wedge RightOf(x, y)) \rightarrow Smaller(x, y))$

(iii) a. $\neg \forall x(Dodec(x) \rightarrow Larger(a, x))$
b. $\forall x\,(Dodec(x) \rightarrow \neg Larger(a, x))$

(iv) a. $\exists x\,(Dodec(x) \wedge \forall y(Cube(y) \rightarrow \neg LeftOf(y, x)))$
b. $\neg \exists x \exists y\,(Cube(x) \wedge Dodec(y) \wedge LeftOf(x, y))$

4.

(i) [6 marks]

1. ∀x (Tet(x) → Large(x))
2. ¬Large(c)
   3. Tet(c)
   4. Tet(c) → Large(c) ✔ ∀ Elim *:1*
   5. Large(c) ✔ → Elim *:3,4*
   6. ⊥ ✔ ⊥ Intro *:2,5*
7. ¬Tet(c) ✔ ¬ Intro *:3-6*

(ii) [6 marks]

1. ∀x (Tet(x) → LeftOf(x, b))
2. ∀y (LeftOf(y, b) → Small(y))
   3. [a] Tet(a)
   4. Tet(a) → LeftOf(a, b) ✔ ∀ Elim: 1
   5. LeftOf(a, b) ✔ → Elim: 3,4
   6. LeftOf(a, b) → Small(a) ✔ ∀ Elim: 2
   7. Small(a) ✔ → Elim: 5,6
8. ∀y (Tet(y) → Small(y)) ✔ ∀ Intro: 3-7

(iii) [6 marks]

1. ∃x (Tet(x) ∨ Cube(x))
2. ∀y ¬Cube(y)
   3. [a] Tet(a) ∨ Cube(a)
   4. ¬Cube(a) ✔ ∀ Elim: 2
   5. Tet(a) ✔ DS: 3,4
   6. ∃x Tet(x) ✔ ∃ Intro: 5
7. ∃x Tet(x) ✔ ∃ Elim: 1,3-6