Philosophy 1102

Answers to Problem Set 8

Total: 50 marks

1. (i) [3 marks]

	Boolean googles	FO goggles
$\forall x\, \neg Tet(x) \rightarrow \exists y\, Small(y)$ $\neg\exists y\, Small(y)$ ------ $\exists x\, Tet(x)$	$P \rightarrow Q$ $\neg Q$ ----------- R	$\forall x\, \neg P(x) \rightarrow \exists y\, Q(y)$ $\neg\exists y\, Q(y)$ ------------------- $\exists x\, P(x)$
TT consequence?	No	
FO consequence?	Yes	
Logical consequence?	(Yes)	

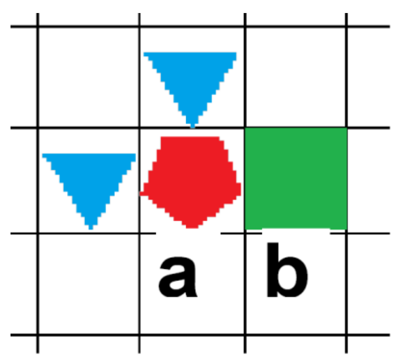
Row of the truth table:

P	Q	R
F	F	F

(ii) [5 marks]

	Boolean googles	FO goggles
∀x (Tet(x) → LeftOf(x, a)) Cube(b) ∧ LeftOf(a, b) ------ ∀y (Tet(y) → LeftOf(y, b))	A B ∧ C --- D	∀x (P(x) → Q(x, a)) R(b) ∧ Q(a, b) ------ ∀y (P(y) → Q(y, b))
TT consequence?	No	
FO consequence?	No	
Logical consequence?	Yes	

Replace LeftOf with Adjoins. (N.B. Loves(x, y) would also work, or anything else that's not transitive.) Then the argument becomes:

T ∀x (Tet(x) → Adjoins(x, a)) T Cube(b) ∧ Adjoins(a, b) ------ F ∀y (Tet(y) → Adjoins(y, b))		Counter-example world: 

(iii) [2 marks]

	Boolean googles	FO goggles
$\neg$ (Smaller(a, b) $\land$ $\exists$x Small(x)) ------ Smaller(a, b) $\rightarrow \neg \exists$x Small(x)	$\neg(A \land B)$ ----- $A \rightarrow \neg B$	$\neg(P(a, b) \land \exists x\, Q(x))$ ------- $P(a, b) \rightarrow \neg \exists x\, Q(x)$
TT consequence?	Yes	
FO consequence?	(Yes)	
Logical consequence?	(Yes)	

(iv) [5 marks]

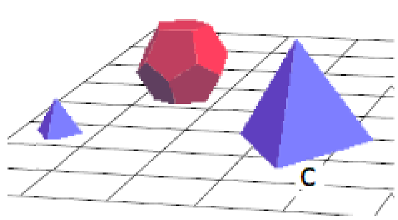
	Boolean googles	FO goggles
$\exists$x $\neg$ Cube(x) $\rightarrow$ Tet(c) ------ Dodec(c) $\rightarrow$ $\forall$x Cube(x)	$P \rightarrow Q$ ----- $R \rightarrow S$	$\exists x \neg P(x) \rightarrow Q(c)$ ------ $R(c) \rightarrow \forall x\, P(x)$
TT consequence?	No	
FO consequence?	No	
Logical consequence?	Yes	

Replace Dodec with Large (or anything else that makes Q(c) and R(c) consistent).
Then the argument becomes:

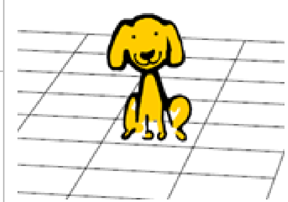
$\exists x \neg Cube(x) \rightarrow Tet(c)$

$Large(c) \rightarrow \forall x\, Cube(x)$

Counter-example world:



2. [2 marks each part]

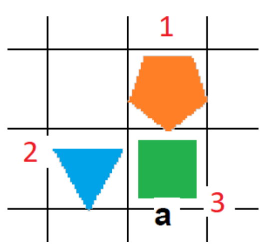
		Logically necessary?	World
(i)	$\exists y(Tet(y) \lor Cube(y)) \leftrightarrow (\exists y\, Tet(y) \lor \exists y\, Cube(y))$	*Yes*	
(ii)	$\exists x\,(Cube(x) \land Large(x)) \quad \leftrightarrow \quad \exists x(Cube(x) \rightarrow Large(x))$ *F* *T*	*No*	
(iii)	$\forall y(Dodec(y) \land Large(y)) \leftrightarrow (\forall y\, Dodec(y) \land \forall y\, Large(y))$	*Yes*	You need a non-cube, and no large cube.

3. [2 marks each, 14 total]

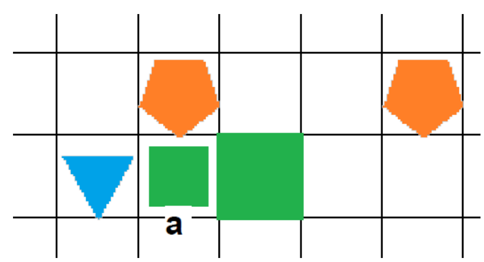
T 1. $\forall x\, \forall y\, ((Small(x) \land Large(y)) \rightarrow FrontOf(x, y))$	1. All the small blocks are in front of all the large blocks.
T 2. $\exists x\, \exists y\, (Cube(x) \land Tet(y) \land Larger(x, y))$	2. There's a cube that is larger than a tetrahedron.
T 3. $\forall x\, \forall y\, ((Cube(x) \land Cube(y)) \rightarrow SameCol(x, y))$	3. All the cubes are in the same column.
T 4. $\neg\forall x\, \forall y\, ((Tet(x) \land Tet(y)) \rightarrow SameCol(x, y))$	4. The tetrahedra aren't all in the same column.
T 5. $\forall x\, \forall y\, ((Cube(x) \land Cube(y) \land x \neq y) \rightarrow \neg SameRow(x, y))$	5. Every cube is in a different row from every *other* cube.
T 6. $\neg\forall x\, \forall y\, ((Tet(x) \land Tet(y) \land x \neq y) \rightarrow \neg SameRow(x, y))$	6. It's not the case that every tetrahedron is in a different row from every *other* tetrahedron.
T 7. $\exists x\, \exists y\, (Tet(x) \land Tet(y) \land x \neq y \land SameSize(x, y))$	7. There are *different* tetrahedra that are the same size.

4.(i) Fill out the satisfaction table below, using Adams' world. [4 marks] Then highlight or draw a ring around the truth value of the whole sentence, and try to see *why* the whole sentence has that truth value.) [1 mark]

x =	y =	$\neg$	$\exists x$	$\forall y$	$(x \neq y$	$\rightarrow$	Adjoins(x, y)))
	1				F	T	F
1	2			F	T	F	F
	3				T	T	T
	1				T	F	F
2	2	F	T	F	F	T	F
	3				T	T	T
	1				T	T	T
3	2			T	T	T	T
	3				F	T	F



Adams' World



McGee's World

(ii) [1 mark each, 3 total]

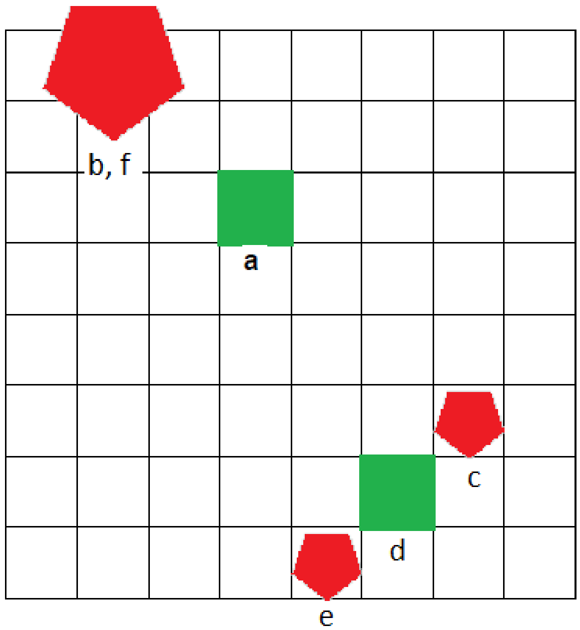
1. $\forall y\ (b \neq y \rightarrow \text{Adjoins}(b, y))$	b adjoins everything else
2. $\exists x\ \forall y\ (x \neq y \rightarrow \text{Adjoins}(x, y))$	There is a thing that adjoins everything else
3. $\neg \exists x\ \forall y\ (x \neq y \rightarrow \text{Adjoins}(x, y))$	Nothing adjoins everything else

Hint: $\forall x\ (\text{InEnglish}(x)) \rightarrow \neg\, \text{ContainsVariables}(x)\ )$

(iii) Do you now understand sentence 3? If so, then say whether the sentence is true or false in McGee's world above. [2 marks]

The sentence is <u>true</u> in McGee's World.

5. [1 mark for each correct object, total 5 marks]



	Sentence	Meaning
1.	∀x((x = a ∨ x = d) ↔ ∃y∃z Between(x, y, z))	a and d (and only a and d) are between 2 things
2.	e = c ↔ a = d	(obvious)
3.	∀x(¬∃y Smaller(y, x) → (x = c ∨ x = e))	∀x(nothing is smaller than x → (x = c ∨ x = e)) (c and e are the only smallest things)
4.	∃x∃y∃z (BackOf(x, y) ∧ BackOf(y, z) ∧ BackOf(z, e))	There are ≥3 things back of e (in 3 different rows)
5.	∀x(x = b → (Dodec(x) ∧ Larger(x, d)))	Everything that's b is a dodec larger than d (I.e. b is a dodec that is larger than d)
6.	(f = b ∨ f = c) ∧ ¬∃x Between(x, b, f)	(f = b ∨ f = c) ∧ nothing is between b and f