Philosophy 1102

Instructor: Richard Johns

Problem Set 8

Hand in answers to the following questions during class on Thursday, March 14.

1. Write out each argument below with Boolean goggles on, then again with FO goggles. Say 'yes' or 'no' to each question: Is it TT con? Is it FO con? Is it Logical con? [*Note*: Remember that TT con ⇒ FO con ⇒ logical con.] Then support your answers by doing what it says below. [15 marks total]

- If the argument is **TT con** then no further work is needed.
- If the argument is **FO con but not TT con** then show that it isn't TT con by giving a counter-example row of the truth table. (Keep the Boolean goggles on for this.)
- If the argument is a **logical consequence, but neither FO con nor TT con** then:
  (a) *Replace the nonsense predicates* with new (meaningful) ones and
  (b) *Draw a counter-example world* for your new argument (i.e. the argument created in part (a)).

(i)

	Boolean googles	FO goggles
$\forall x \neg Tet(x) \rightarrow \exists y\, Small(y)$ $\neg\exists y\, Small(y)$ ------ $\exists x\, Tet(x)$		
TT consequence?		
FO consequence?		
Logical consequence?		

(ii)

	Boolean googles	FO goggles
$\forall x\ (\text{Tet}(x) \rightarrow \text{LeftOf}(x, a))$ $\text{Cube}(b) \wedge \text{LeftOf}(a, b)$ ------ $\forall y\ (\text{Tet}(y) \rightarrow \text{LeftOf}(y, b))$		
TT consequence?		
FO consequence?		
Logical consequence?		

(iii)

	Boolean googles	FO goggles
¬ (Smaller(a, b) ∧ ∃x Small(x)) ------ Smaller(a, b) → ¬ ∃x Small(x)		
TT consequence?		
FO consequence?		
Logical consequence?		

(iv)

	Boolean googles	FO goggles
∃x ¬Cube(x) → Tet(c) ------ Dodec(c) → ∀x Cube(x)		
TT consequence?		
FO consequence?		
Logical consequence?		

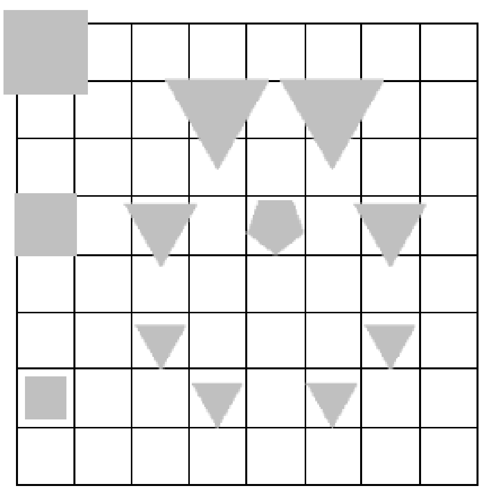
2. For each of the following sentences,

(a) Say whether it is logically necessary or not, and

(b) (If it is logically necessary then you're done.) If it is *not* logically necessary, then draw a world where the biconditional is false (i.e. draw a *single* world in which the sentences on either side of the '↔' have different truth values). [2 marks each, 6 total]

		Logically necessary?	World
(i)	$\exists y(\text{Tet}(y) \lor \text{Cube}(y)) \leftrightarrow (\exists y\ \text{Tet}(y) \lor \exists y\ \text{Cube}(y))$		
(ii)	$\exists x\ (\text{Cube}(x) \land \text{Large}(x)) \leftrightarrow \exists x(\text{Cube}(x) \to \text{Large}(x))$		
(iii)	$\forall y(\text{Dodec}(y) \land \text{Large}(y)) \leftrightarrow (\forall y\ \text{Dodec}(y) \land \forall y\ \text{Large}(y))$		

3. Translate the sentences provided below into FOL. All the sentences are true in the world (Finsler's world) shown. *Note that every answer should be either a double existential* $\exists x \exists y$, *or a double universal* $\forall x \forall y$, *or the negation of one of these sentence types.*
[2 marks each, 14 total]



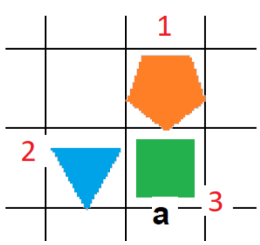
1. All the small blocks are in front of all the large blocks.
2. There's a cube that is larger than a tetrahedron.
3. All the cubes are in the same column.
4. The tetrahedra aren't all in the same column.
5. Every cube is in a different row from every *other* cube.
6. It's not the case that every tetrahedron is in a different row from every *other* tetrahedron.
7. There are *different* tetrahedra that are the same size.

[*Hint*: The italics in these sentences are a hint!]

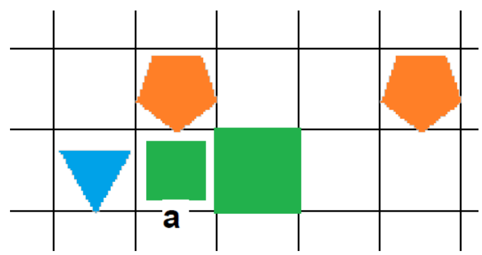
4. The purpose of this question is to practice understanding a difficult sentence:
$\neg \exists x \, \forall y \, (x \neq y \rightarrow \text{Adjoins}(x, y))$.

(i) Fill out the satisfaction table below, using Adams' world. [4 marks] Then highlight or draw a ring around the truth value of the whole sentence, and try to see *why* the whole sentence has that truth value.) [1 mark]

x =	y =	$\neg$	$\exists$x	$\forall$y	(x $\neq$ y	$\rightarrow$	Adjoins(x, y)))
1	1						
	2						
	3						
2	1						
	2						
	3						
3	1						
	2						
	3						



Adams' World



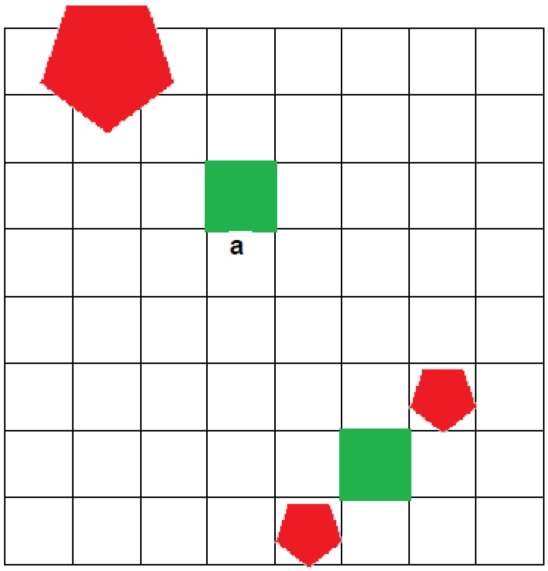
McGee's World

(ii) Another method is to translate the sentence into English gradually, "from the inside out". Notice sentences 1 and 2 below are gradually building up to the hard one (3). Translate all three sentences, *as simply as possible*, bearing in mind that your translations for 1 and 2 should each help you to translate the next sentence.
[*Hint*: English sentences don't contain variables like 'x' and 'y'.] [1 mark each, 3 total]

1. $\forall y \, (b \neq y \rightarrow \text{Adjoins}(b, y))$
2. $\exists x \, \forall y \, (x \neq y \rightarrow \text{Adjoins}(x, y))$
3. $\neg \exists x \, \forall y \, (x \neq y \rightarrow \text{Adjoins}(x, y))$

(iii) Do you now understand sentence 3? If so, then say whether the sentence is true or false in McGee's world above. [2 marks]

5. Using the sentences provided, which are all true in the world shown, attach the names **b**, **c**, **d**, **e** and **f** to the objects in the world below. *Note that one object has two different names*. (You should also use the method from Qu. 4 of reading complex sentences like #3 from the inside out.) [1 mark for each correct object, total 5 marks]



	Sentence	Hint
1.	$\forall x((x = a \lor x = d) \leftrightarrow \exists y \exists z\ \text{Between}(x, y, z))$	Tells you which one is d
2.	$e = c \leftrightarrow a = d$	Tells you whether or not e = c
3.	$\forall x(\neg \exists y\ \text{Smaller}(y, x) \rightarrow (x = c \lor x = e))$	Tells you which two blocks are c and e, but not which is which.
4.	$\exists x \exists y \exists z\ (\text{BackOf}(x, y) \land \text{BackOf}(y, z) \land \text{BackOf}(z, e))$	Tells you which one is e.
5.	$\forall x(x = b \rightarrow (\text{Dodec}(x) \land \text{Larger}(x, d)))$	Tells you which one is b
6.	$(f = b \lor f = c) \land \neg \exists x\ \text{Between}(x, b, f)$	Tells you which one is f