

Answers to Problem Set 7

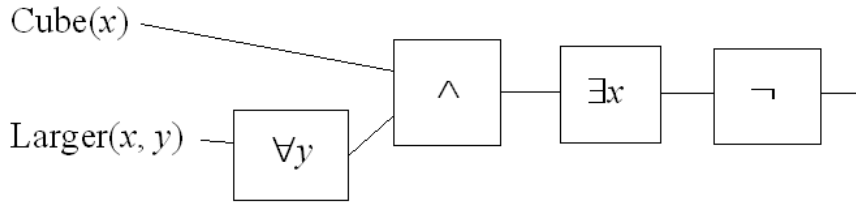
Total: 50 marks

1. [1 mark each, 8 total]

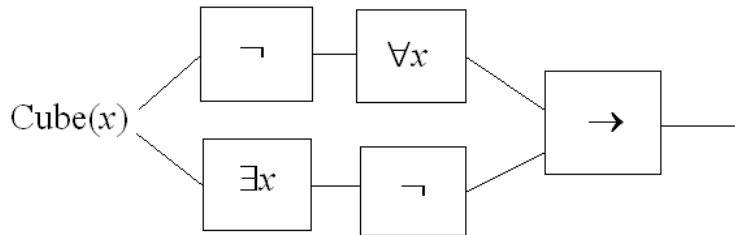
		Wff?	Sentence?
(i)	$\forall a (\text{Medium}(a) \rightarrow \text{Cube}(a))$	No	(No)
(ii)	$\text{Larger}(\exists, b) \rightarrow \text{Tet}(b)$	No	(No)
(iii)	$\exists t (\text{Dodec}(t) \rightarrow \text{Tet}(t))$	Yes	Yes.
(iv)	$\text{Cube}(a) \rightarrow \text{Tet}(x) \rightarrow \text{Tet}(y)$	No	(No)
(v)	$\exists x (\text{Dodec}(x) \wedge \text{Large}(\forall))$	Yes	No
(vi)	$\forall z (\exists w \text{Large}(w) \rightarrow \text{Cube}(z))$	Yes	Yes
(vii)	$\forall v \text{Cube}(v) \rightarrow \text{Large}(\forall)$	Yes	No
(viii)	$\forall x (\text{Large}(\text{Cube}(x)) \rightarrow \text{Small}(y))$	No	(No)

2. [2 marks each, 8 total]

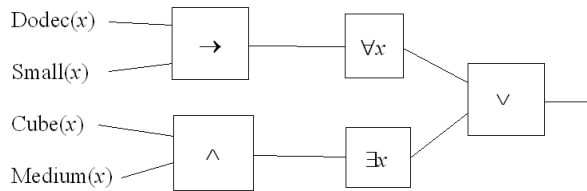
(i) **Negation Sentence** [Question was: $\neg \exists x(\text{Cube}(x) \wedge \forall y \text{Larger}(x, y))$]



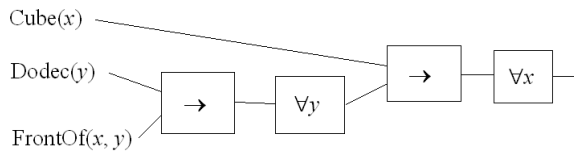
(ii) **Conditional Sentence** [Question was: $\forall x \neg \text{Cube}(x) \rightarrow \neg \exists x \text{Cube}(x)$]



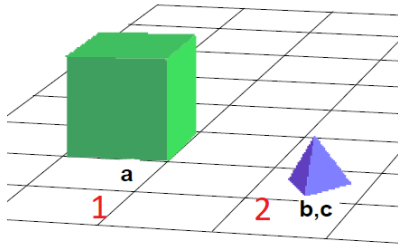
(iii) **Disjunction Sentence** $\forall x (\text{Dodec}(x) \rightarrow \text{Small}(x)) \vee \exists x (\text{Cube}(x) \wedge \text{Medium}(x))$
[question below]



(iv) **Universal Sentence** $\forall x(\text{Cube}(x) \rightarrow \forall y(\text{Dodec}(y) \rightarrow \text{FrontOf}(x, y)))$
[question below]



3. For each sentence below, fill in the satisfaction table to determine whether the sentence is true or false in the world provided. [1 mark for each table + 1 for each truth value = 6 total]



(i) $\forall x (\text{Cube}(x) \rightarrow x = a)$ Truth value **T** [1 mark]

x =	$\forall x$	(Cube(x)	\rightarrow	(x = a	\vee	x = c))
1	T	T	T	T	T	F
2		F	T	F	T	T

(ii) $\forall x (x = a \vee x = b)$ Truth value **T** [1 mark]

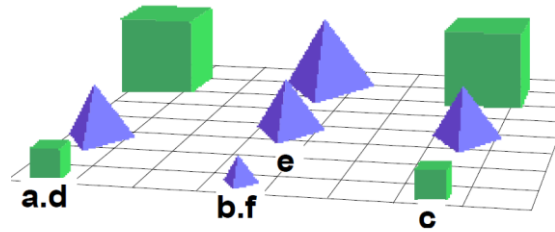
x =	$\forall x$	(x = a	\vee	x = b)
1	T	T	T	F
2		F	T	T

(iii) $\exists x (\text{Smaller}(x, a) \wedge x \neq b)$ Truth value **F** [1 mark]

x =	$\exists x$	(Smaller(x, a)	\wedge	x \neq b)
1	F	F	F	T
2		T	F	F

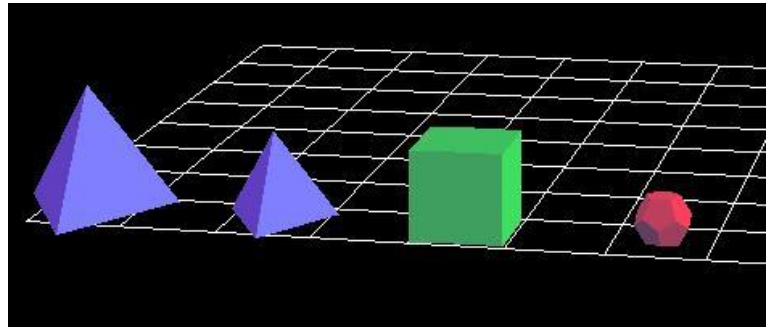
4. [1 mark for each truth value, 8 total]

- T 1. $\exists x (x \neq a \wedge x \neq b \wedge x \neq c \wedge x \neq d \wedge x \neq e)$
-
- T 2. $\forall x (x = a \rightarrow x = d)$
-
- F 3. $\exists x (\text{Between}(x, c, a) \wedge x \neq b)$
-
- T 4. $\forall x (\text{Between}(x, c, a) \rightarrow x = b)$
-
- F 5. $\forall x ((\text{Tet}(x) \wedge \text{Medium}(x)) \rightarrow x = e)$
-
- T 6. $\forall x (x = e \rightarrow (\text{Tet}(x) \wedge \text{Medium}(x)))$
-
- T 7. $\forall x ((\text{Tet}(x) \wedge \text{Small}(x)) \leftrightarrow x = b)$
-
- T 8. $\exists y (y \neq e \wedge \text{SameRow}(y, e))$



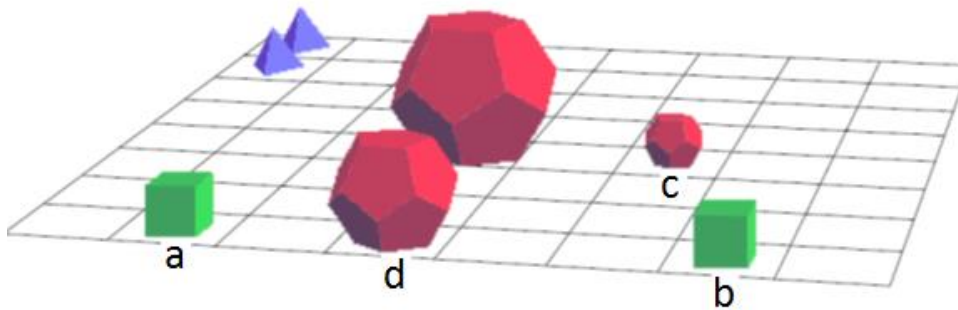
5. [5 marks for world]

- T 1. $\exists x (\text{Tet}(x) \wedge \text{Large}(x))$
-
- T 2. $\exists x (\text{Tet}(x) \wedge \text{Medium}(x))$
-
- T 3. $\exists x (\text{Cube}(x) \wedge \neg \text{Small}(x))$
-
- T 4. $\exists y (\text{Dodec}(y) \wedge \neg \text{Large}(y))$
-
- T 5. $\forall x (\text{Cube}(x) \rightarrow \text{Medium}(x))$
-
- T 6. $\forall x (\text{Dodec}(x) \rightarrow \text{Small}(x))$
-
- T 7. $\forall x (\text{Tet}(x) \rightarrow \neg \text{Small}(x))$
-
- T 8. $\forall y (\text{Cube}(y) \rightarrow \neg \text{Tet}(y))$



6. [1 mark each, 5 total]

1. All the tetrahedra are small. $\forall x (\text{Tet}(x) \rightarrow \text{Small}(x))$
2. Every large thing is a dodecahedron. $\forall y (\text{Large}(y) \rightarrow \text{Dodec}(y))$
3. Some dodecahedron is small. $\exists x (\text{Dodec}(x) \wedge \text{Small}(x))$
4. Some dodecahedron is neither large nor small. $\exists x (\text{Dodec}(x) \wedge \neg \text{Large}(x) \wedge \neg \text{Small}(x))$
5. No tetrahedron is medium. $\neg \exists x (\text{Tet}(x) \wedge \text{Medium}(x))$



7. Translate the five sentences below into FOL. If correct, all five sentences will be true in the world from Question 6. [2 marks each]

1. Some small dodecs are in back of d.

$$\exists x (\text{Small}(x) \wedge \text{Dodec}(x) \wedge \text{BackOf}(x, d))$$

2. Only dodecs are medium.

$$\forall x (\text{Medium}(x) \rightarrow \text{Dodec}(x))$$

3. Every tetrahedron is both left of and the same size as c.

$$\forall x (\text{Tet}(x) \rightarrow (\text{LeftOf}(x, c) \wedge \text{SameSize}(x, c)))$$

4. No large dodec is in the same row as b.

$$\neg \exists x (\text{Large}(x) \wedge \text{Dodec}(x) \wedge \text{SameRow}(x, b))$$

5. Every dodec that's in the same row as a is medium.

$$\forall x ((\text{Dodec}(x) \wedge \text{SameRow}(x, a)) \rightarrow \text{Medium}(x))$$