Philosophy 1102

Instructor: Richard Johns

Problem Set 10

Hand in your answers during class on Thursday, March 28.

[For these proofs, use only the rules provided in the handout “Rules of F^{+}”.]

1. Show that the argument below is FO con by giving a formal proof. [6 marks]

$$\begin{array}{|l} \forall x\ \text{Cube}(x) \\ \forall y\ \text{Small}(y) \\ \hline \forall z\ (\text{Small}(z) \land \text{Cube}(z)) \end{array}$$

2. Show that the argument below is FO con by giving a formal proof. [6 marks]

$$\begin{array}{|l} \exists y\ \text{Cube}(y) \lor \exists z\ \text{Small}(z) \\ \hline \exists x(\text{Cube}(x) \lor \text{Small}(x)) \end{array}$$

3. Show that the argument below is FO con by giving a formal proof. [6 marks]

$$\begin{array}{|l} \exists x \exists y\ (\text{Cube}(x) \land \text{Cube}(y) \land \neg \text{SameRow}(x, y)) \\ \hline \neg \forall x \forall y\ ((\text{Cube}(x) \land \text{Cube}(y)) \rightarrow \text{SameRow}(x, y)) \end{array}$$

[N.B. you can eliminate the $\exists x$ and $\exists y$ quantifiers in one step if you like, since they’re touching.]

4. Show that the argument below is FO con by giving a formal proof. [6 marks]

$$\begin{array}{|l} \neg \exists x\ \text{Cube}(x) \\ \hline \forall x \neg \text{Cube}(x) \end{array}$$

5. Show that the argument below is *not* FO con, by doing the following: [5 marks]

(a) Write the argument as it appears through FO goggles
(b) Provide a counter-example set of predicates to replace the nonsense predicates used in part (a), and
(c) Draw a counter-example world, for the new argument (with the new predicates)

| ∃x(SameShape(x, a) ∧ Cube(x))
|―――
| ¬(Tet(a) ∨ Dodec(a))

6. By giving a formal proof, show that the argument in Qu. 5 becomes FO con when suitable shape axioms (see below) are added as premises. (Cite the axioms as 'A1', 'A2', etc.) [7 marks]

Shape Axioms

A1. ¬∃x (Cube(x) ∧ Tet(x))	A6. ∀x ∀y ((Dodec(x) ∧ Dodec(y)) → SameShape(x, y))
A2. ¬∃x (Tet(x) ∧ Dodec(x))	A7. ∀x ∀y ((Tet(x) ∧ Tet(y)) → SameShape(x, y))
A3. ¬∃x (Dodec(x) ∧ Cube(x))	A8. ∀x ∀y ((SameShape(x, y) ∧ Cube(x)) → Cube(y))
A4. ∀x (Tet(x) ∨ Dodec(x) ∨ Cube(x))	A9. ∀x ∀y ((SameShape(x, y) ∧ Dodec(x)) → Dodec(y))
A5. ∀x ∀y ((Cube(x) ∧ Cube(y)) → SameShape(x, y))	A10. ∀x ∀y ((SameShape(x, y) ∧ Tet(x)) → Tet(y))

7. Show that the argument below is FO con by giving a formal proof. [7 marks]

| ∃x (Dog(x) ∧ Lazy(x))
| ∀x∀y ((Dog(x) ∧ Dog(y)) → x = y)
|―――
| ∃x (Dog(x) ∧ ∀y (Dog(y) → x = y) ∧ Lazy(x))

8. Show that the argument below is FO con by giving a formal proof. [7 marks]

[Make sure you understand what the first premise says. An important question then is: Does Celine love Celine?]

| ∀x(∃y Loves(x, y) → Loves(x, celine))
| Loves(celine, bill)
| ¬Loves(alice, alice)
|―――
| alice ≠ celine