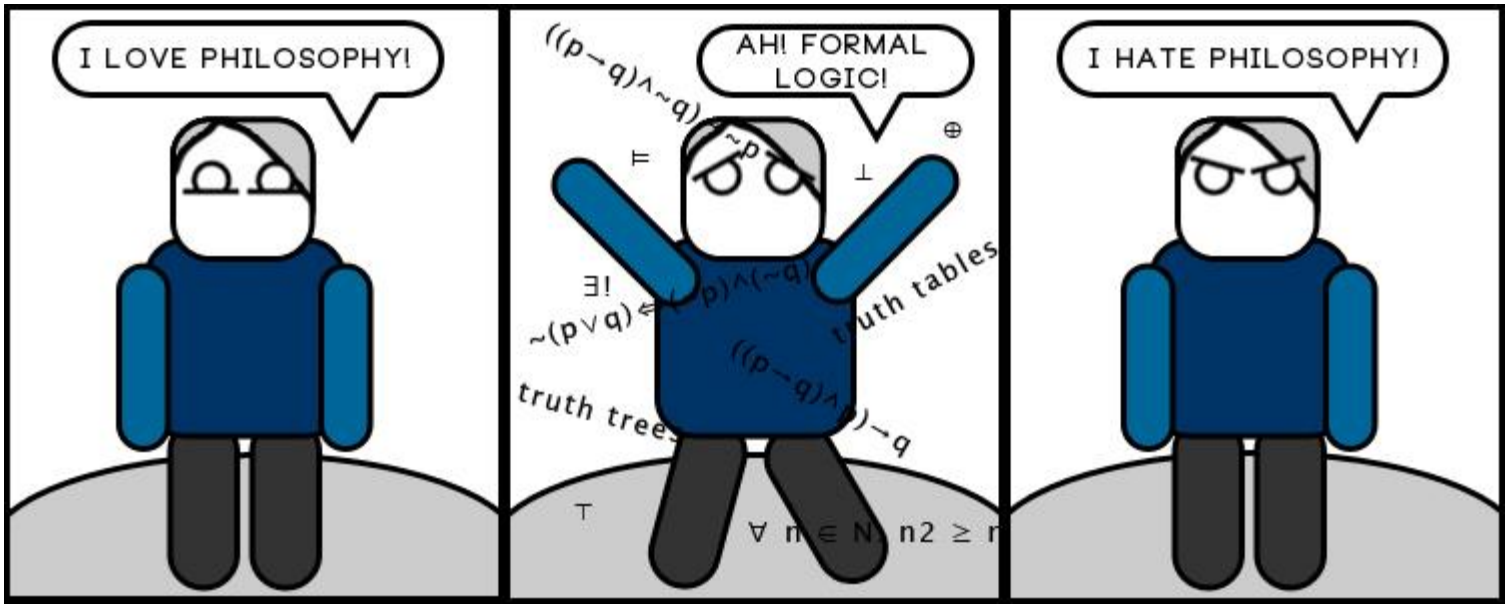
Table 3: Some Valid Formulas of Propositional Calculus	
law	formula
Law of identity	$p \equiv p$
Law of double negation	$p \equiv \sim\sim p$
Law of excluded middle	$p \vee \sim p$
Law of noncontradiction	$\sim(p \cdot \sim p)$
De Morgan laws	$(p \cdot q) \equiv \sim(\sim p \vee \sim q)$
	$(p \vee q) \equiv \sim(\sim p \cdot \sim q)$
Commutative laws	$(p \vee q) \equiv (q \vee p)$
	$(p \cdot q) \equiv (q \cdot p)$
Associative laws	$[(p \vee q) \vee r] \equiv [p \vee (q \vee r)]$
	$[(p \cdot q) \cdot r] \equiv [p \cdot (q \cdot r)]$
Law of transposition	$(p \supset q) \equiv (\sim q \supset \sim p)$
Distributive laws	$[p \cdot (q \vee r)] \equiv [(p \cdot q) \vee (p \cdot r)]$
	$[p \vee (q \cdot r)] \equiv [(p \vee q) \cdot (p \vee r)]$
Law of permutation	$[p \supset (q \supset r)] \equiv [q \supset (p \supset r)]$
Law of syllogism	$(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$
Law of importation	$[p \supset (q \supset r)] \supset [(p \cdot q) \supset r]$
Law of exportation	$[(p \cdot q) \supset r] \supset [p \supset (q \supset r)]$

Formal Logic

Mind your Ps and Qs!


I LOVE PHILOSOPHY!
AH! FORMAL LOGIC!
truth tables
truth tree
I HATE PHILOSOPHY!


Philosophy majors thinking they can avoid math
Predicate logic
Pedagogical Memes on Philosophical Themes

Argument vs. Explanation

- Arguments and explanations often have a similar structure.

- They both have what we might (vaguely) call a ‘basis’ and a ‘result’. They might both take the form:

“Since <basis>, we have <result>”,

or: “<result>, because <basis>”.

Argument vs. Explanation

- In an argument, the basis is the premises, which are already believed. The result is the conclusion, which the argument tries to make us believe.
- An explanation proposes a possible *cause* for some observed event. The result is the observed event, the basis is the proposed cause, which isn't observed in most cases.

Plain assertion

- Sometimes we simply *assert* (say) something. We aren't trying to argue for it, or explain why it happened. We're simply saying that it is the case.

Argument, explanation or assertion?

- I never enjoyed playing sports at school. I don't like watching pro sports on TV either. In fact, I don't like sports, period.

- Just 3 assertions.

Argument, explanation or assertion?

- There is a God. I believe this because that's how I was raised. I went to Sunday school every week, and read the Bible all the time.
- Explanation

I was raised to believe in God *caused* I believe in God

Argument, explanation or assertion?

- At the present rate of consumption, global oil production will peak in about 5 years. And we're sure not going to reduce consumption in the near future. So we'd better start developing solar power, windmills, and other "alternative" energy sources pretty soon.

- Argument

Argument, explanation or assertion?

- The abortion issue is blown out of all proportion. How come we don't hear nearly as much about the evils of the Pill? After all, a lot more potential people are "killed" by the Pill than by abortion.

- Argument (possibly a poor one!)

Argument, explanation or assertion??

- The official account of how the World Trade Center towers collapsed is very fishy. For one thing, they do not attempt to explain the pools of glowing molten metal found in the rubble piles.

- Argument (Note that one can argue for some claim on the basis that it provides a good explanation for something.)

Argument, explanation or assertion?

- Why is a sodium flame yellow? Because of the ionisation energy of sodium atoms. Photons of this energy have a wavelength that we see as yellow.

  Explanation

  Sodium atoms have ionisation energy XYZ

  Photons of energy XYZ appear yellow to us

  causes

  Sodium flames are yellow

Argument, explanation or assertion?

- Why should you believe that the earth is warming up? Because the concentration of CO_2 is up, and in the past this has always meant higher temperatures.

- Argument

Argument, explanation or assertion?

- The dinosaurs died out as a result of global climate change. This was most likely due a large meteor impact, that would have put a lot of dust in the upper atmosphere, blocking the sun.

- Explanation

Argument, explanation or assertion?

- The dinosaurs died out as a result of global climate change. The fossil record shows that it happened very suddenly, and there were no other animals around that could have forced them into extinction. Mammals didn't really spread until the dinosaurs were already gone. It had to be severe climate change, as nothing else could have done it.
- Argument

The meanings of ‘because’

- The word ‘because’ can be used to state an argument or an explanation. I.e. it expresses either logical consequence or cause and effect.

  <conclusion>, because <premise>

  <effect> because <cause>

1. In each of the following sentences, say whether the word ‘because’ expresses a relation of logical consequence, or of cause and effect. State the premise and conclusion, or cause and effect, as appropriate.

- God exists, because otherwise life would be meaningless.
- The river bank collapsed because of the heavy rain last week.
- Abortion is not wrong, because a woman should be able to control her own body.
- The moon *was* full last night because I saw it!

Standard Form

The ‘standard form’ of an argument is as follows:

Premise 1

Premise 2

… (etc.)

Conclusion

- God can’t be both perfectly good and all-powerful. After all, if God were perfectly good he would want to eliminate all evil. And if God were all-powerful he would do whatever he wanted. But evil certainly exists!

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1. If God were perfectly good he would want to eliminate all evil.
2. If God were all-powerful he would do whatever he wanted.
3. Evil exists.

God isn't both perfectly good and all-powerful.

- Labour is the basis of all property, since nothing can be obtained without someone working. From this it follows that a man naturally owns what he makes by his own hands, so that capitalism is an unjust economic system.

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nothing can be obtained without someone working

⇓

Labour is the basis of all property

⇓

a man naturally owns what he makes by his own hands

⇓

capitalism is an unjust economic system

- Labour is the basis of all property, since nothing can be obtained without someone working. From this it follows that a man naturally owns what he makes by his own hands, so that capitalism is an unjust economic system.

1. Nothing can be obtained without someone working.

Capitalism is an unjust economic system

- All philosophers are smokers, and most smokers are unfit. So most philosophers are unfit.

  All philosophers are smokers

  Most smokers are unfit

  Most philosophers are unfit

  (Is it *valid*?)

Part 2

Validity and Truth

Validity and Truth

- Recall that a *valid* argument is one where the premises have the maximum possible degree of positive relevance to the conclusion.
- In other words, the premises of a valid argument provide *conclusive proof* of the conclusion.
- A consequence of this fact is that a valid argument *cannot* have true premises and a false conclusion. ***No such situation can even be consistently imagined***.

Counter-example “world”

fit

fit

fit

T All philosophers are smokers
T Most smokers are unfit

F Most philosophers are unfit

unfit

unfit

unfit

unfit

Validity isn't enough …

All mammals have gills.

Humans are mammals

Humans have gills

-- but in formal logic, validity is all we look at.

Arguments with unstated premises or conclusion (enthymemes)

- “Whales suckle their young because whales are a type of mammal”

Whales are a type of mammal

(All mammals suckle their young)

∴ Whales suckle their young

What’s the unstated premise?

Unstated conclusion

- “Fred is a politician. Politicians are corrupt. Need I say more?”

  Fred is a politician

  Politicians are corrupt

  (Fred is corrupt)

- “I don’t want to criticise my husband’s cooking. But he is an Englishman, and he isn’t Jamie Oliver.”

  My husband is an Englishman

  My husband isn’t Jamie Oliver

  (My husband is a bad cook)

- “Of course Janet makes a lot of money. She’s a lawyer, isn’t she?”

  Janet is a lawyer

  (Lawyers make a lot of money)

  Janet makes a lot of money

Yeah right, cyclists should be allowed to treat stop signs as ‘yield’ signs. Even as the law stands, a lot of cyclists blow past stop signs as if they didn’t exist, scattering pedestrians in all directions. Do you think that telling cyclists they don’t have to stop will make them *more* careful?

1. A lot of cyclists already run stop signs dangerously
2. (Telling cyclists they don’t have to stop will make them **less** careful)

(Cyclists should **not** be allowed to treat stop signs as ‘yield’ signs)

Review

1. Say whether each of the following passages contains an *argument*, an *explanation*, or a simple *assertion*.

(i) The City's building permit process is just crazy. So many rules! I feel like I'm helplessly ensnared in a sticky web of regulations.

(ii) The City's building permit process is just crazy. I think that every time something bad happens, someone invents a new rule to stop that happening again. These bureaucrats are so risk-averse, so afraid of litigation, that they keep adding rules "just to be on the safe side". They never consider the stress this creates for ordinary people, just trying to fix their houses.

(iii) The City's building permit process is just crazy. A comparison with other municipalities across North America shows that we have almost twice as many forms to fill out as the average, and wait times have tripled over the last five years.

2. Put the following arguments into standard form. I.e. List the premises, draw a horizontal line beneath them, then write the conclusion underneath. Omit any sentence that is neither the conclusion nor a premise. **In this exercise *do not* add any unstated premises**.

(i) Beer is living proof that God loves us and wants us to be happy. (Benjamin Franklin)

(ii) If we don't reduce our carbon emissions drastically in the next 20 years, the results will be devastating. We already know that increased carbon dioxide levels are responsible for most of the warming during the last century. And there are nasty feedback mechanisms, so that warmer temperatures will cause more CO_2 to be released, causing more warming ... It follows that the status quo will lead to huge temperature increases, which we know would be very harmful to the planet.

(iii) Computers can't really think -- they only simulate thought. The easiest way to see this is to consider the fact that there's nothing mysterious about how computers work. To an engineer, the operation of a computer is as transparent as that of a toaster! But I've already shown that we can never understand thought itself. The nature of thinking will always be a mystery to us.

Part 3

Categorical Logic

Aristotelian (Categorical) Logic

- Aristotle identified four very common sentence *forms*, i.e. patterns or structures.
  - Some arguments can be shown to be valid by examining the forms of the premises and the conclusion.

(A) All S is P (*or* Every S is P)

(E) No S is P

(I) Some S is P

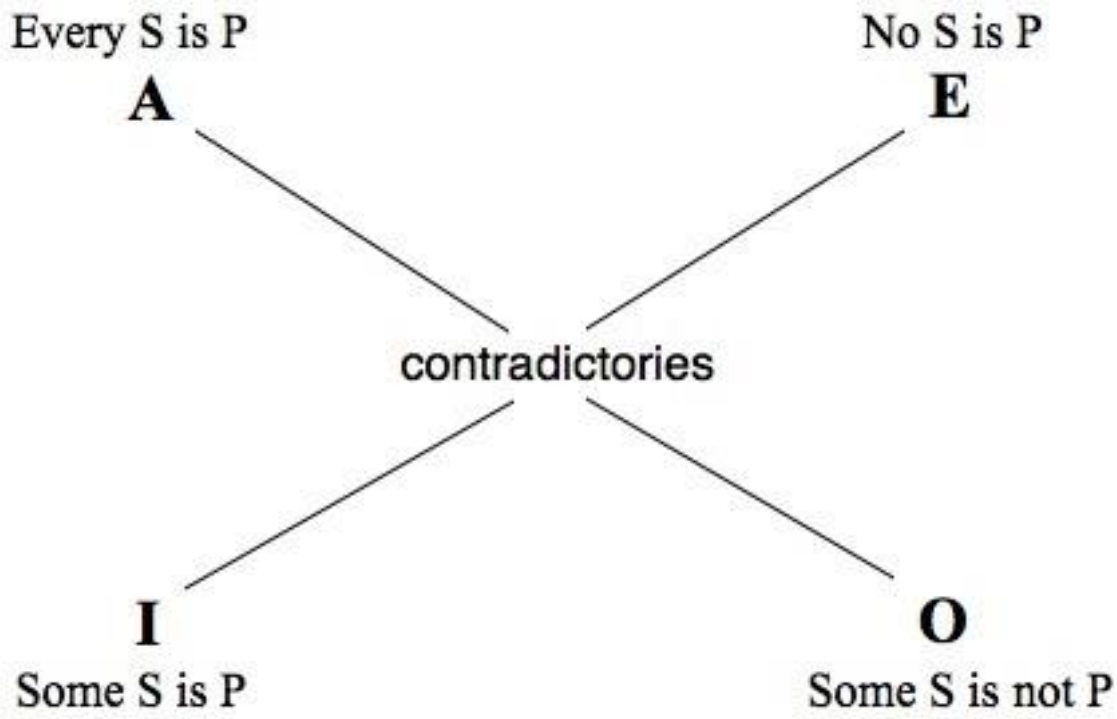
(O) Some S is not P

- N.B. In logic, **"some" = *at least one*.**

- So it's *true* that:
  - Some moons orbit the earth
  - Some Langara professors are human
  - Some Popes have been Catholic

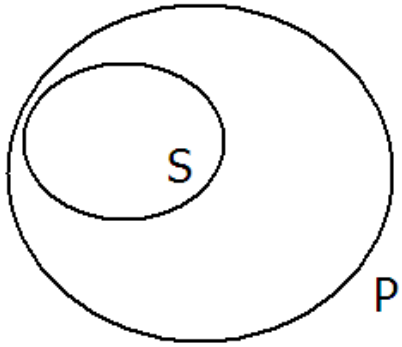
    (Etc.)

The Square of Opposition

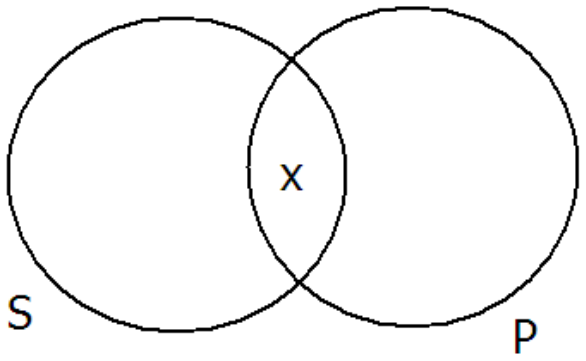


Venn Diagrams

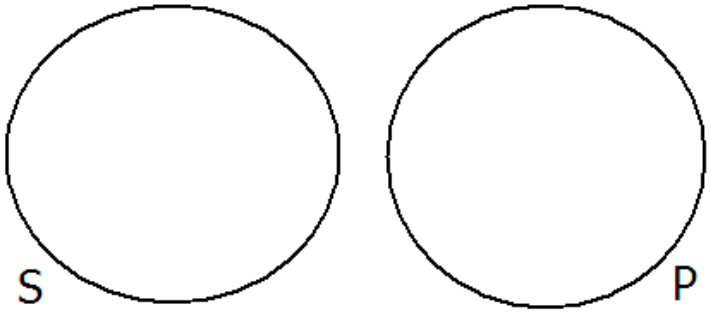
Every S is P



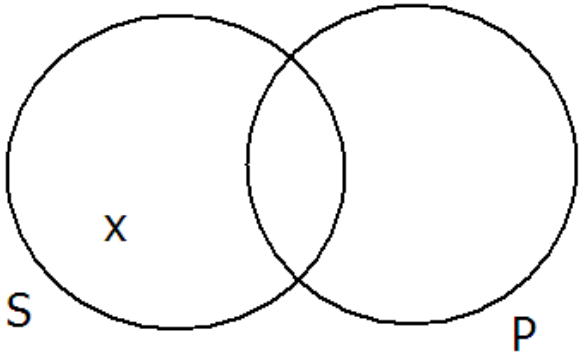
Some S is P



No S is P

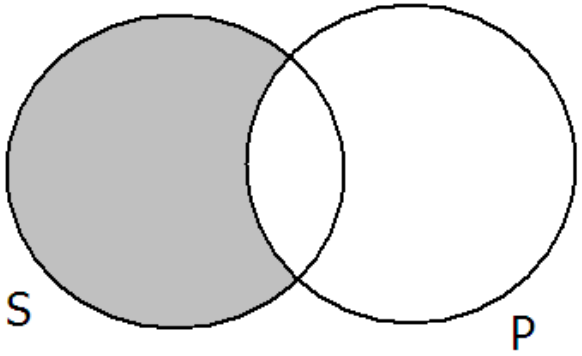


Some S is not P

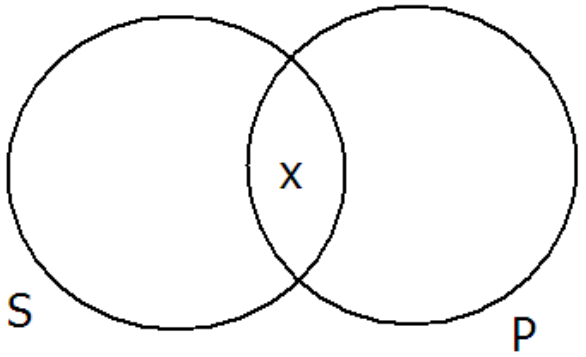


Or better …

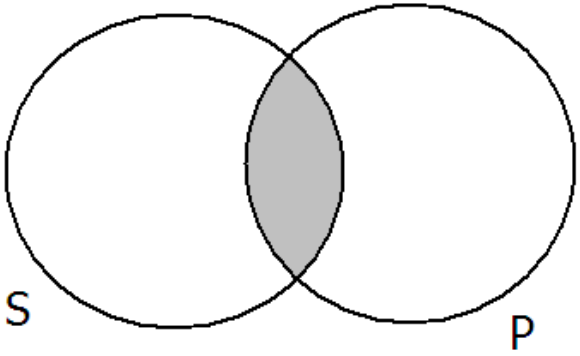
Every S is P



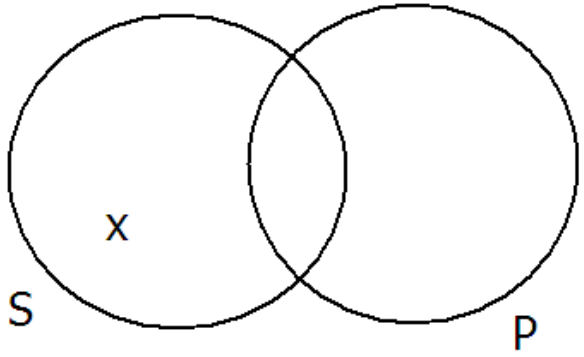
Some S is P



No S is P



Some S is not P



= empty

3. Paraphrase the following sentences into one of Aristotle's forms (A, E, I and O) from the square of opposition.

(i) If you're 25 years old and riding the bus, then you're a failure. (*Margaret Thatcher*)

All 25-yr-old bus riders are failures. (All S are P)

(ii) There aren't any honest politicians.

No honest thing is a politician. (No S is P)

(iii) Not all professors are dull.

Some profs are not dull. (Some S is not P)

(iv) There is such a thing as a flying car.

Some flying thing is a car. (Some S is P)

Keep it as simple as possible:

- Avoid *negative* predicates where possible.
  - E.g. “All students are poor”
  - Not “no student is non-poor”,
  - or “All non-poor things are non-students”.

- Avoid *compound* predicates where possible.
  - E.g. write “some students are poor”,
  - Not “some things are poor students”

Is it valid?

- “No machines are conscious, but some animals are conscious, so no animals are machines.

  No M are C

  Some A are C

  No A are M

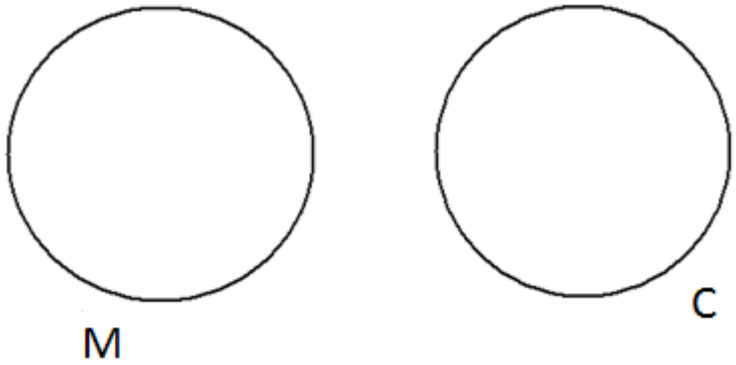
How can we figure it out?

1. No M is C
2. Some A is C

No A is M

We might read the first premise as saying “there’s no overlap between M and C”.

And picture it like this:

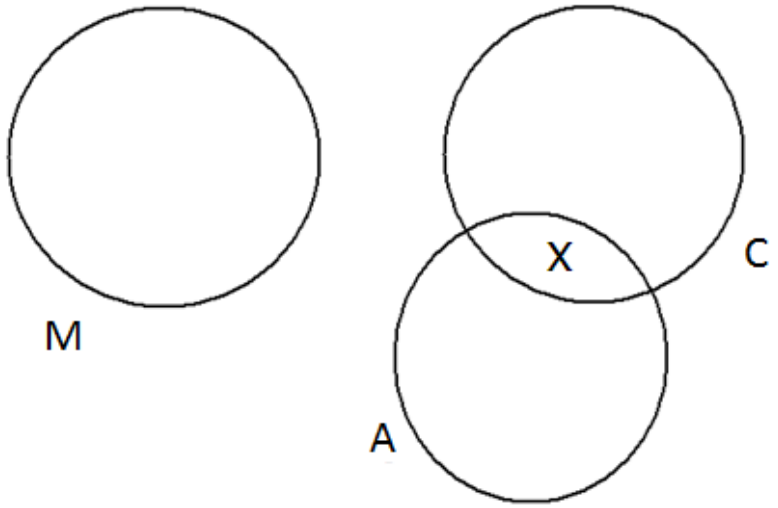


How can we figure it out?

1. No M is C
2. Some A is C

No A is M

Now, what about A? P2 says there's *some* overlap between A and C. So shall we draw it like this?

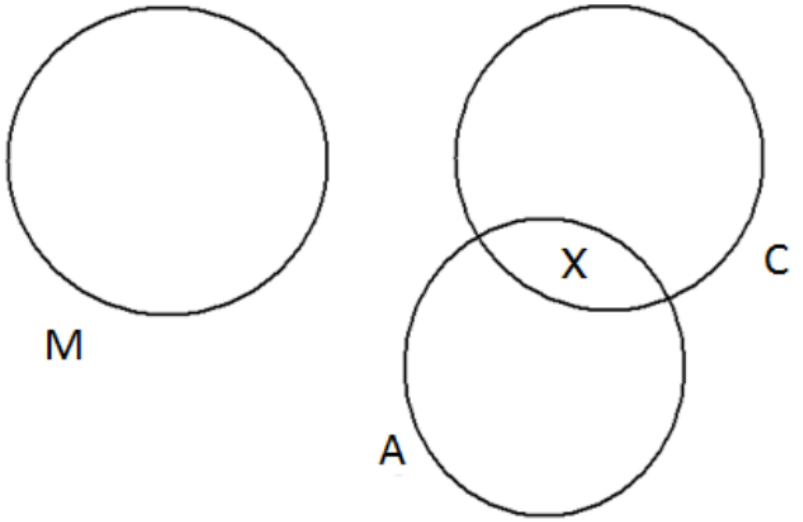


How can we figure it out?

1. No M is C
2. Some A is C

No A is M

In this “world”, both premises are true, and the conclusion is true as well. So the argument is valid?

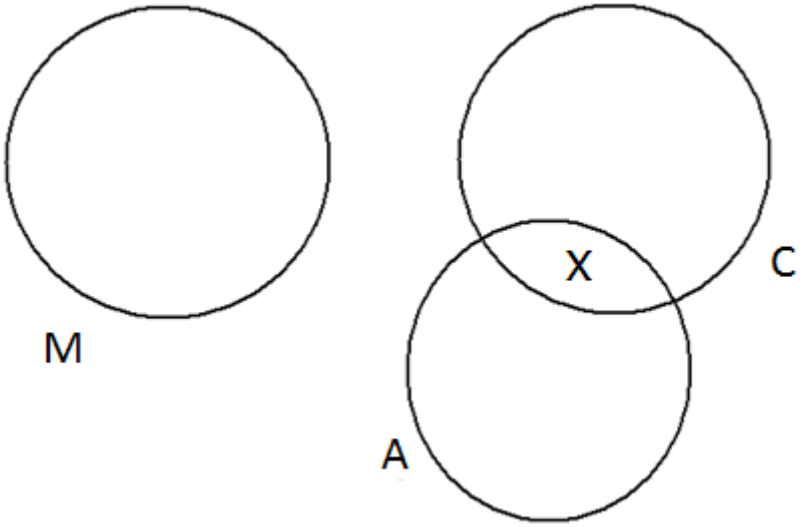


How can we figure it out?

1. No M is C
2. Some A is C

No A is M

NO! Because there may be *other* "worlds" in which the premises are both true, yet the conclusion is *false*.

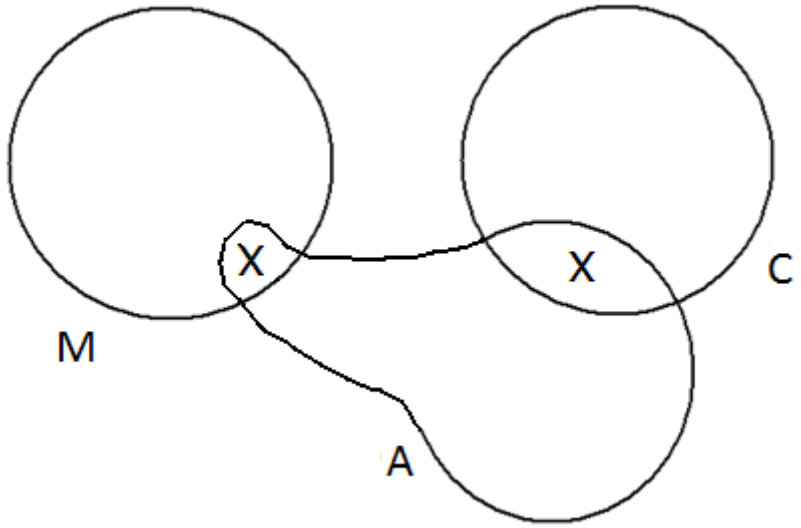


How can we figure it out?

1. No M is C
2. Some A is C

No A is M

NO! Because there may be *other* "worlds" in which the premises are both true, yet the conclusion is *false*. (*Like this, for example*.)

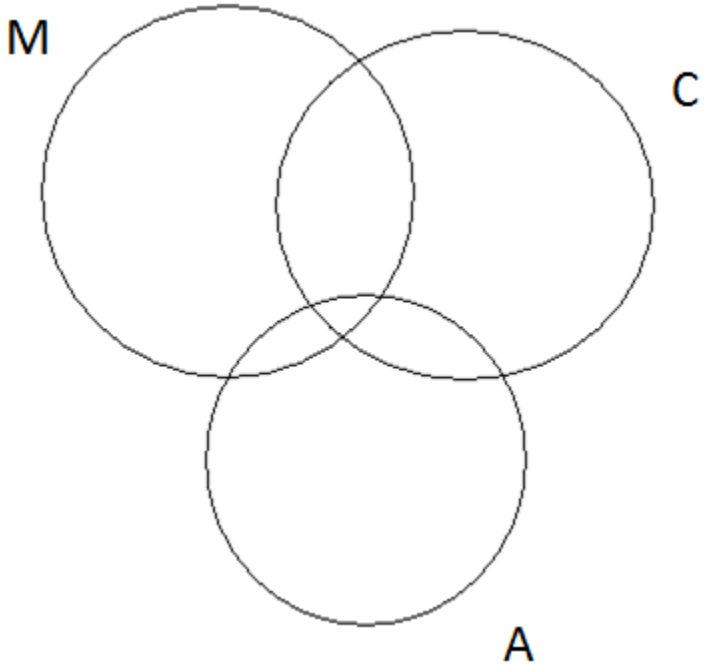


A “fool-proof” method

With 3 properties, there are 8 possible combinations of them, as shown in this “Venn diagram”. We can just put shading and x’s where needed.

1. No M is C
2. Some A is C

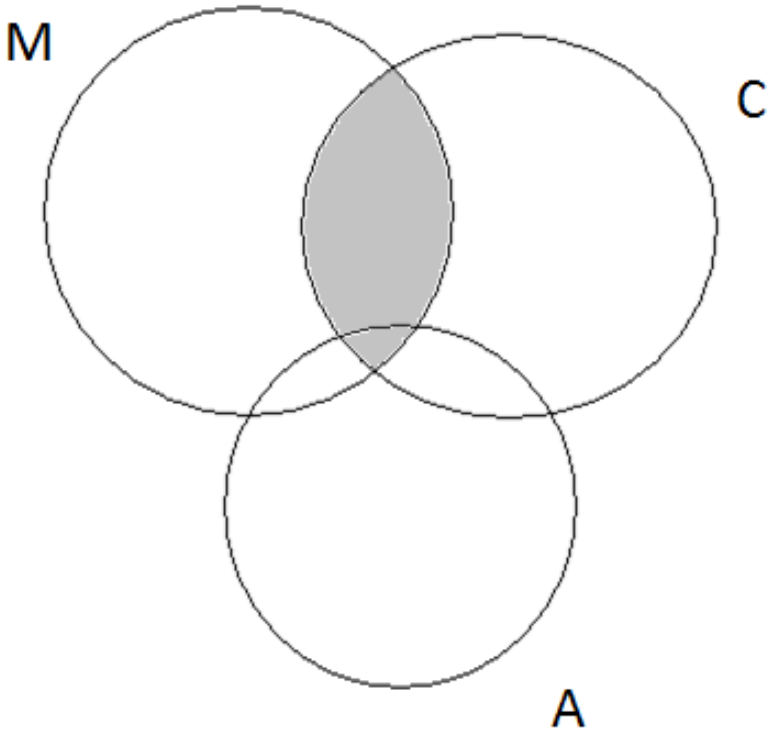
No A is M



1. No M is C

1. No M is C
2. Some A is C

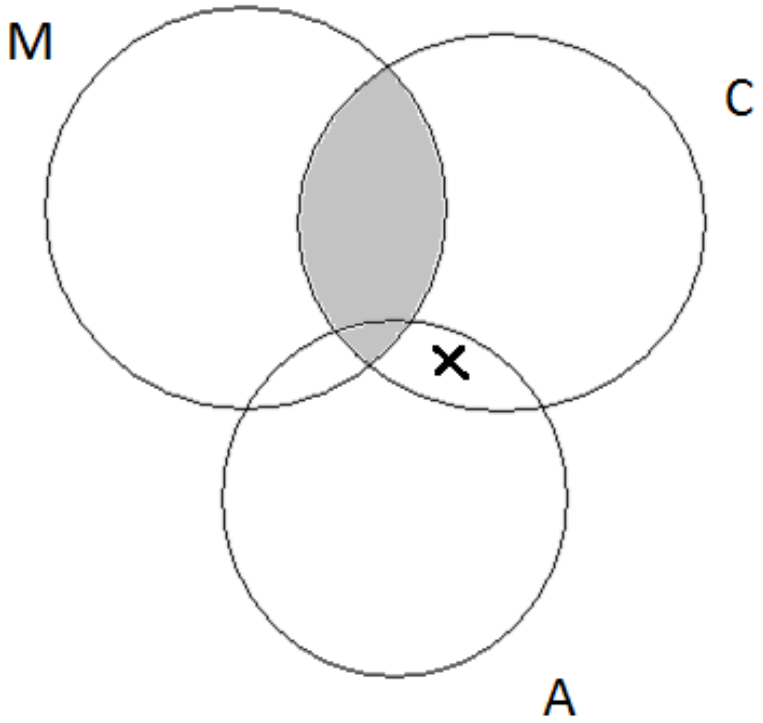
No A is M



2. Some A is C

1. No M is C
2. Some A is C

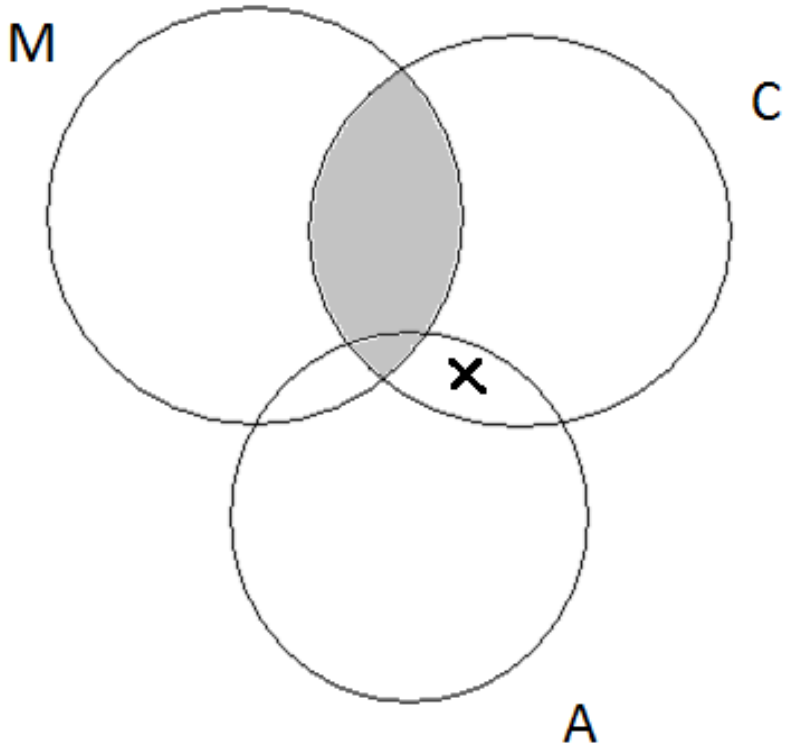
No A is M



The question now is whether the diagram tells us that the conclusion (No A is M) is true. Does it?

1. No M is C
2. Some A is C

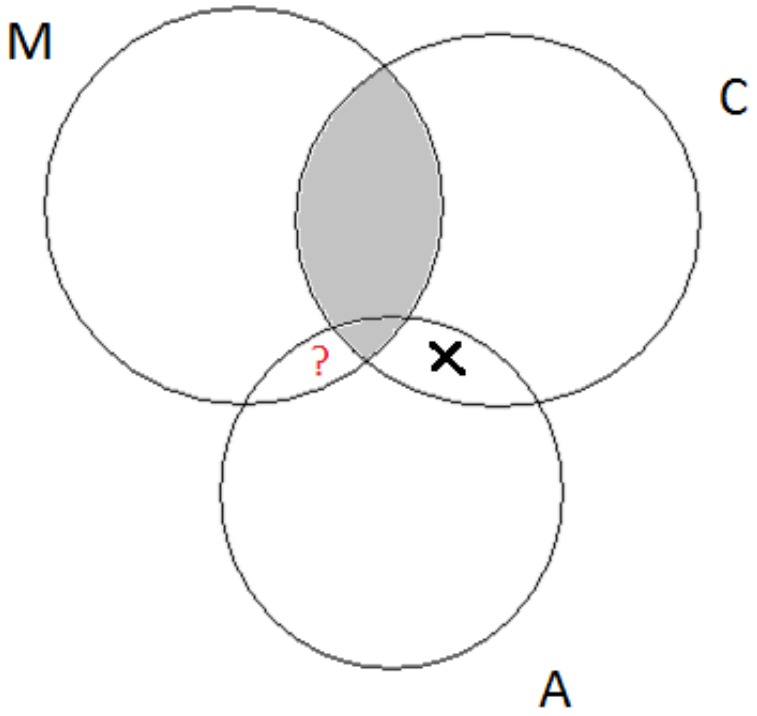
No A is M



No, it doesn't tell us this. It *allows* the conclusion to be true, but doesn't *guarantee* it. There might well be some object that is **M, A and not C**, as shown by '?'.

1. No M is C
2. Some A is C

No A is M



T 1. No M is C

T 2. Some A is C

F No A is M

The argument is *invalid*.

Animal
Machine
Not conscious

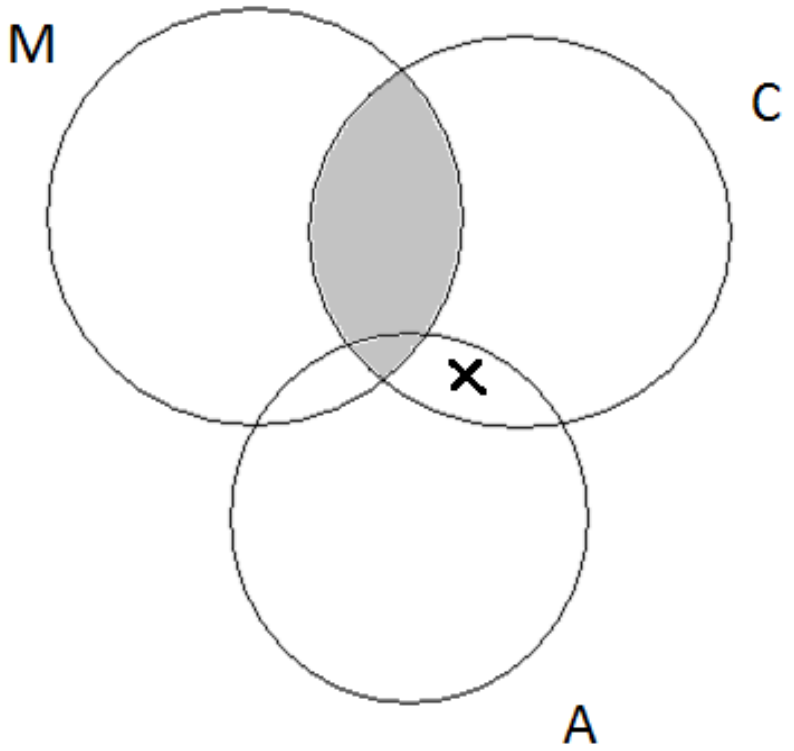
Machine
Not conscious

Animal
Conscious

If the conclusion were changed to "*some* animals are not machines", then is the resulting argument valid?

1. No M is C
2. Some A is C

Some A is not M



- For each of the following arguments, *say whether or not it is valid*. (I.e. don't worry about whether or not the premises are acceptable.)

  If it is valid then provide a **proof**, *or* draw a **Venn diagram** showing what the premises tell us.

  If it's invalid then give a **counter-example world** (a possible situation where the premises are all true but the conclusion is false).

(i) All politicians are ruthless, and no one without loyal friends can be a politician. Of course politicians exist! It follows that some ruthless people have loyal friends.

Is it valid?

- (If you're not sure, then draw the Venn diagram, or perhaps try to draw a counter-example world.

- First, put everything into Aristotelian form.

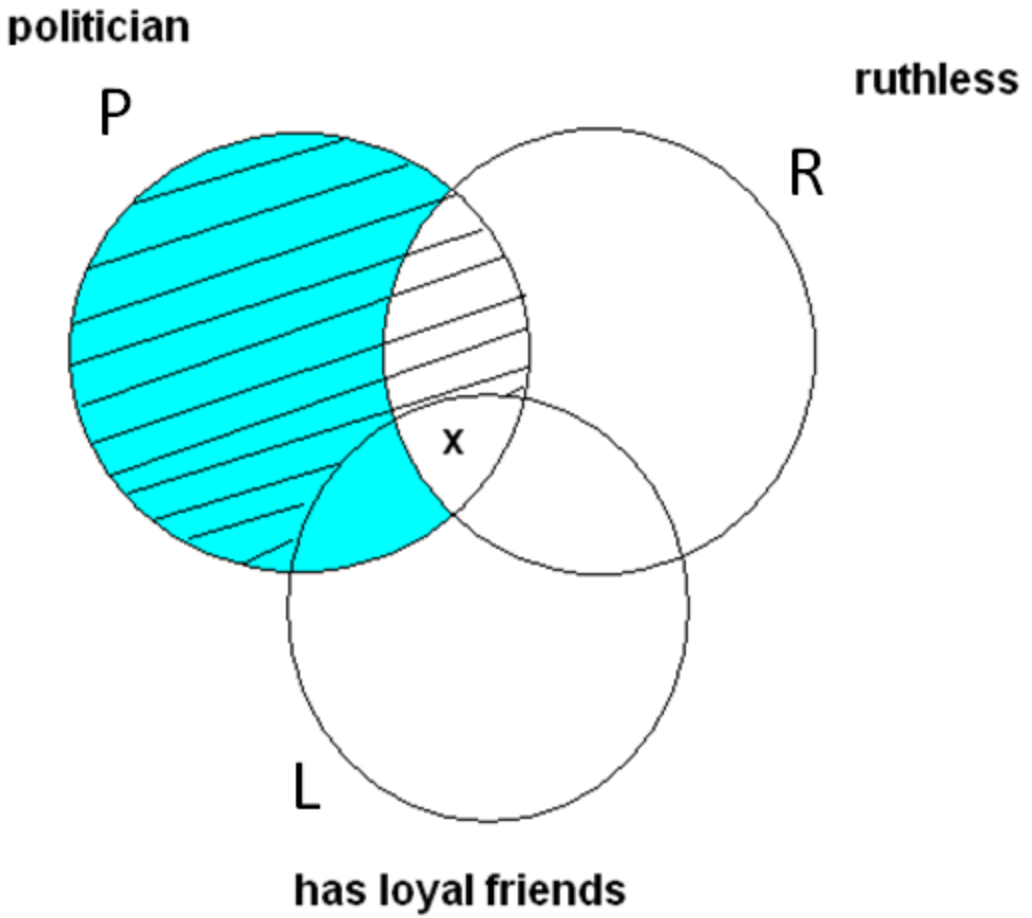
- All politicians are ruthless
- All P are R
- no one without loyal friends can be a politician.
- All P are L
- Of course politicians exist!
- Some T are P
- Some ruthless people have loyal friends.
- Some R are L

All P are R

All P are L

Some T are P

Some R are L



Qu. Does the diagram *tell* us that Some R are L?

Yes. So it's valid.

Since it's valid, you can do a proof instead of a Venn diagram.

Proof: We are told (P3) that politicians exist, so let 'Fred' name such a politician. Since all politicians are ruthless (P1) it follows that Fred is ruthless. Further, Fred must have loyal friends, according to P2 (since no one without such friends is a politician). Hence Fred is a ruthless person with loyal friends, so that some such people exist.■

(ii) No genius is modest, but no hillbilly is a genius. So all hillbillies must be modest.

Invalid

hillbilly
modest
not genius

hillbilly
not modest
not genius

genius
not modest

(iii) All dogs are hairy animals, and all dogs are also 4-legged. So we see that all hairy animals are 4-legged.

- **Invalid.**

hairy animal
8 legged

hairy animal
dog
4 legged

hairy animal
dog
4 legged

(iv) All lazy people are highly efficient. So clearly, no professors are lazy, as no professors are highly efficient.

All lazy people are highly efficient

No professors are highly efficient.

No professors are lazy

Valid? You can try to make a counter-example world.

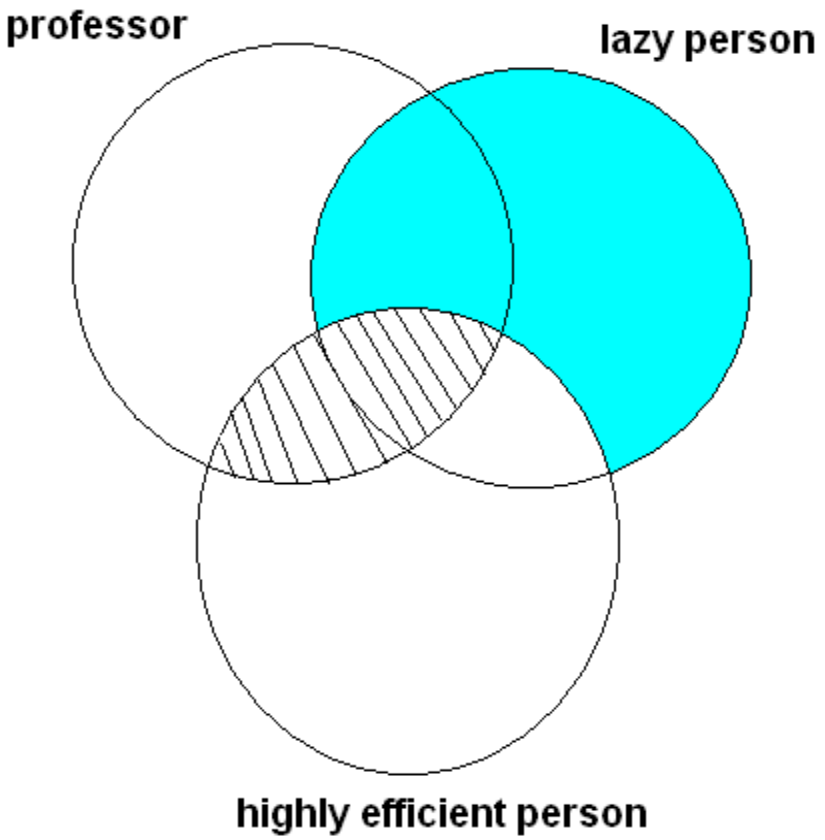
Lazy
Professor
Efficient
Not efficient!!

Contradiction!
So, no counter-example exists

Proof: We will argue by *reductio ad absurdum*. Let us assume that the conclusion is false, i.e. that some professor is lazy. Let Fred be such a lazy professor. Using P1, and the fact that Fred is lazy, we then infer that Fred is highly efficient. But from P2, since Fred is a professor, we infer that Fred is not highly efficient. This is a contradiction, and hence the conclusion follows by *reductio*. ■

(iv) All lazy people are highly efficient
No professors are highly efficient.

No professors are lazy



Part 4

Conditional sentences

Conditional Sentences

If A, *then* B

(antecedent) (consequent)

- What does this really say?
  - Does it say *A*? Does it say *B*?

- How is it related to “A, if B”, “A only if B”, “A unless B”, etc.?

- Do we really want to look at conditionals?
- They’re hard.
- Maybe we should cut them from the course?

I guess we have to cover conditionals then …

- ***If A then B*** is an assertion of ***B***, that is "in effect", or "switched on", only in the circumstance that ***A*** is true.

- If A is false, in fact, then the statement is "silent", and not asserted at all.

- E.g. suppose I say, "if you clean my room then I will give you $5".
  - You *do* clean my room: = "I will give you $5".
  - You *don't* clean my room: = nothing, null and void.

Believing conditionals

- What does it mean to *believe* a conditional?

- E.g. suppose engineer believes that *if* there's an earthquake, *then* this bridge will collapse. (If *E* then *C*). What does she *believe*? E? C?

- She believes *C*, but not in her actual state of knowledge *K*. She believes *C* in the "expanded" state of knowledge *K+E*.

Other forms of the conditional

"B, *if* A" $\Leftrightarrow$ if A then B

"A, *unless* B" $\Leftrightarrow$ If $\neg$B then A

"A *only if* B" $\Leftrightarrow$ If $\neg$B then $\neg$A

$\Leftrightarrow$ If A then B

"A *if and only if* B" $\Leftrightarrow$ If B then A and if A then B

“unless …”

- Why do we sometimes add an “unless …” clause onto the end of a sentence?
  - Think about the “epistemic situation”

- Suppose someone asks if you’ll go hiking with them on Saturday. You really want to go, so you say “yes”.
  - But then you remember that your boss said she *might* call you to do an extra shift on Saturday, paying you time and a half, and the extra cash would come in handy.
- So you modify your response to:

  “I’ll go hiking, **unless** my boss calls me in”

“I’ll go hiking, **unless** my boss calls me in”

- Is it:

  If my boss calls me in, I won’t go hiking ?

  or

  If my boss doesn’t call, I’ll go hiking ?

“I’ll go hiking, **unless** my boss calls me in – in which case I’ll probably take the shift.”

Conversational Implication

- We are familiar with the fact that every statement has logical consequences.

- E.g. “This man was born in Vancouver” has the consequence “this man was born in Canada.”

- Conversational implication isn’t like this. With conversational implication, you infer things from the *fact that the person said it*, and also that they *didn’t* say other things.

Conversational Implication

- Conversational implications are meanings that aren't strictly stated in the sentence.
- Some conversational implications follow from the statement, using the "rule of honest communication":

Give *all* the relevant information that you have.

Conversational Implication

- E.g. “damning with faint praise”

  “UBC is the best university in Point Grey”

  “How do you like my home-made wine?”

-- “It has a very unique flavour.”

Conversational Implication

- E.g. “protesting too much”

  “What were you doing in the basement just now?”

-- “I wasn’t drinking your whisky. Definitely not!”

Conversational Implication and Conditionals

- Suppose there are two brothers, Jim and Luke. Their mother hands Jim $5, with instructions that it is to be given to Luke, as it is owed to him.
- Jim is an obedient boy, and will certainly do this.

- But Jim is also crafty, and says to his brother,

  "*if you clean my room for me* then I will give you this crisp $5 bill"

- Is Jim *lying*? (Is he being deceitful?)

Conversational Implication and Conditionals

- If one believes that *B*, and this is relevant, honesty requires that we simply assert *B*.

- By asserting "If *A* then *B*", we're suggesting that we're at least unsure of *B*. Otherwise, people will wonder, what's the point of the "If A" restriction?

Withholding relevant information

- By saying "*if you clean my room for me* then I will give this $5 bill", rather than simply: "I will give you this crisp $5 bill", Jim is withholding relevant information from his brother.

- It's similar to the case where I tell someone: "You have won either $5000 or $1", when I know full well that it is $1.

Conversational Implication

- So, since we presume honesty when people us talk to us,

If A then B has the (conversational) implication that:

If ¬A then (probably) ¬B (‘¬’ = *not*)

“Only if”

- The general function of “only” is to *exclude other cases*.

- “This lounge is for the use of faculty and staff”
- “This lounge is for the use of faculty and staff *only*”

- “You will pass if you study”
- “You will pass *only if* you study”

Conversational Implication

- A father tells a child “You can have ice cream only if you first eat all of your broccoli.”
- The child eats the broccoli, and asks for ice cream.
- The father replies that there is no ice cream in the house, unfortunately.
- The child claims that the father lied. Did he? *Technically*?

Such implication works through people *presuming* honesty in the speaker, thinking, in this case:

"If I can't have ice cream, whatever I do, then Daddy would have just said so."

- For each of the following sentences, write down anything that is not strictly stated, but is suggested by conversational implication.

(i) Everyone other than Fred passed the exam.

(ii) If your GPA drops below 3.6 then you will lose your scholarship.

(iii) Did you get an 'A' on your exam? Well, I can tell you that you *passed* at least.

(iv) A fever is nothing to worry about, unless you also have a bad sore throat.

(v) Not *all* people are reincarnated.

(iii) My brother is, shall we say, “tall, dark, and ... dark”.

(iv) Do I like my mother in law? Oh gosh, is that the time? I must be going!

(v) You will get a promotion only if you land the Yamaha account.

Inferences involving conditionals

- Many arguments have a conditional premise. E.g.

  If I have a cavity, then I need to see a dentist.

  I have a cavity

  ∴ I need to see a dentist

Is this one *valid*?

- What about this one?

If Fred has measles, then he has a fever.

Fred has a fever

$\therefore$ Fred has measles

- What about this one?

If Fred has measles, then he has a fever.

Fred *doesn't* have a fever

∴ Fred *doesn't* have measles

- What about this one?

If Fred is late today, then he will be fired

Fred is *not* late today

∴ Fred will *not* be fired

Modus ponens ✓

(Affirming the antecedent)

If A then B

A

∴ B

Affirming the consequent ✘

If A then B

B

∴ A

Modus Tollens ✓

(Denying the consequent)

If A then B

¬B

∴ ¬A

Denying the antecedent ✘

If A then B

¬A

∴ ¬B

If A, *then* B

(antecedent) (consequent)

- For each of the following arguments identify the *type* of inference (modus ponens, affirming the consequent, modus tollens, denying the antecedent, or a disjunctive argument) and say whether or not it is deductively *valid*. [Note that valid conclusions are *conclusively proved* by the premises, not just supported to some extent.]

(i) Of course I have a soul. A purely physical being, lacking a soul, cannot be conscious, and I know I'm conscious!

$\neg$ soul $\rightarrow$ $\neg$ conscious

I'm conscious!

I have a soul.

- MT, valid

Modus Tollens ✓
(Denying the consequent)

If A then B
$\neg$B

$\therefore$ $\neg$A

N.B. It might not look like the pattern fits, but the key feature is that the 2nd premise *contradicts* the consequent. (And the conclusion contradicts the antecedent.)

(ii) It's clear that the recession is over. If the recession is over we will see increases in building permit applications, and that's happening right now.

recession over $\rightarrow$ increases

increases

---

recession over

AC, invalid

Affirming the consequent ×

If A then B
B

$\therefore$ A

(iv) Jen isn't an expert on Canadian history. Someone with a Ph.D. on the subject is an expert, of course, but Jen doesn't have a Ph.D.

Ph.D $\rightarrow$ Expert

$\neg$ Ph.D

$\neg$ Expert

DA, invalid

Denying the antecedent ×

If A then B

$\neg$A

$\therefore$ $\neg$B

(v) Assuming I am a genius, way ahead of my time, I'm bound to get laughed at. And people do laugh at me, which proves that I am indeed a genius.

Genius $\rightarrow$ Laughed at

Laughed at

Genius

AC, invalid

Affirming the consequent ×

If A then B
B

$\therefore$ A

(vi) Unless I win the lottery tomorrow, I won't be able to pay the rent. And I sure won't be winning it (as I can't even afford to buy a ticket!) Therefore I won't be able to pay the rent.

$\neg$ Win $\rightarrow$ $\neg$ Pay rent

$\neg$ Win

$\neg$ Pay rent

MP, valid

Modus ponens ✓

(Affirming the antecedent)

If A then B

A

$\therefore$ B