Philosophy 1102
Introduction to Logic

Instructor: Richard Johns

Problem Set 7

Hand in answers to the following questions during class on Thursday, March 7.

1. For each of the strings of symbols below, say whether or not it is a *wff* of FOL, and whether or not it is a *sentence* of FOL. *Non-wffs require no further answer*. If it is a wff, however, then draw an arrow from each bound variable to the quantifier that binds it, and draw a ring around any free variables, as shown in the example below. [1 mark for each sentence = 8 total]

		wff?	**sentence?**
E.g.	$\forall x(\text{Cube}(y) \rightarrow \forall y\ \text{Larger}(x, y))$	Yes	No
(i)	$\forall c\ (\text{Medium}(c) \rightarrow \text{Tet}(a))$		
(ii)	$\text{Larger}(\exists, b) \rightarrow \text{Tet}(b)$		
(iii)	$\exists t\ (\text{Dodec}(t) \rightarrow \text{Tet}(t))$		
(iv)	$\text{Cube}(a) \rightarrow \text{Tet}(x) \rightarrow \text{Tet}(y)$		
(v)	$\exists x\ (\text{Dodec}(x) \wedge \text{Large}(y))$		
(vi)	$\forall z\ (\exists w\ \text{Large}(w) \rightarrow \text{Cube}(z))$		
(vii)	$\forall v\ \text{Cube}(v) \rightarrow \text{Large}(v)$		
(viii)	$\forall x\ (\text{Large}(\text{Cube}(x)) \rightarrow \text{Small}(y))$		

2. (a) Translate each of the following FOL sentences into a circuit diagram, and translate each diagram into an FOL sentence. [1 mark each]

(b) Identify the *type* of each sentence (i.e. the main logical operator, which is either a connective or a quantifier). Thus the type of sentence may be 'universal', 'existential', 'conjunction', 'disjunction, 'negation', 'conditional, or 'biconditional'. [1 mark each]

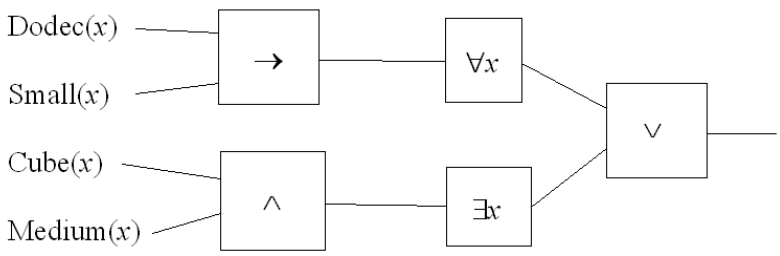
(i) $\neg \exists x\,(\text{Cube}(x) \wedge \forall y\,\text{Larger}(x, y))$ *Diagram*:

Sentence type: _____

(ii) $\forall x \neg \text{Cube}(x) \rightarrow \neg \exists x\,\text{Cube}(x)$ *Diagram*:

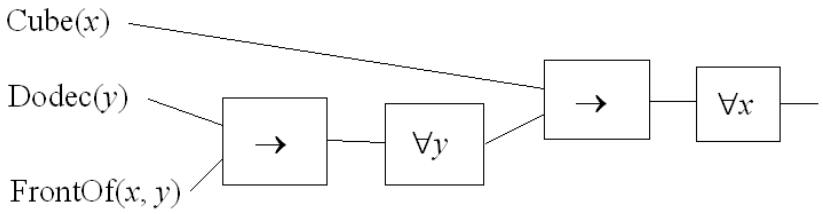
Sentence type: _____

(iii)



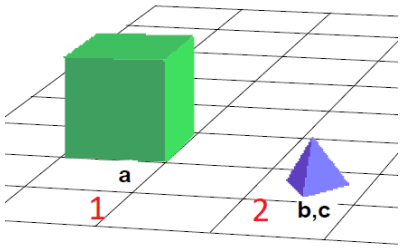
Sentence: _____ *Sentence type*: _____

(iv)



Sentence: _____ *Sentence type*: _____

3. For each sentence below, fill in the satisfaction table to determine whether the sentence is true or false in the world provided. [1 mark for each table + 1 for each truth value = 6 total]



(i) $\forall x\,(Cube(x) \rightarrow (x = a \vee x = c))$ Truth value _____ [1 mark]

x =	∀x	(Cube(x)	→	(x = a	∨	x = c))
1						
2						

(i) $\forall x\,(x = a \vee x = b)$ Truth value _____ [1 mark]

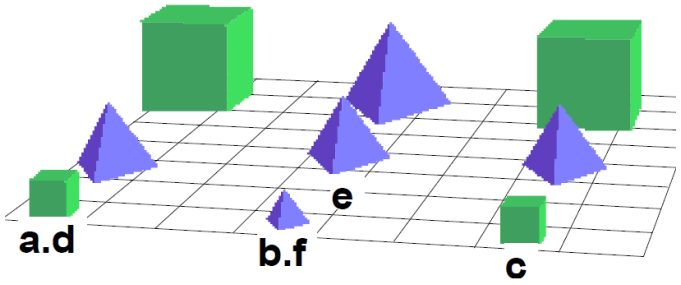
x =	∀x	(x = a	∨	x = b)
1				
2				

(iii) $\exists x\,(Smaller(x, a) \wedge x \neq b)$ Truth value _____ [1 mark]

x =	∃x	(Smaller(x, a)	∧	x ≠ b)
1				
2				

4. Evaluate each of the sentences below in the world provided. (I.e. just say whether each sentence is **T** or **F** in that world.) [1 mark for each truth value]
[If you're unsure of an answer, you could create a satisfaction table – or imagine creating one.]

1. $\exists x\ (x \neq a \wedge x \neq b \wedge x \neq c \wedge x \neq d \wedge x \neq e)$
2. $\forall x\ (x = a \rightarrow x = d)$
3. $\exists x\ (\text{Between}(x, c, a) \wedge x \neq b)$
4. $\forall x\ (\text{Between}(x, c, a) \rightarrow x = b)$
5. $\forall x\ ((\text{Tet}(x) \wedge \text{Medium}(x)) \rightarrow x = e)$
6. $\forall x\ (x = e \rightarrow (\text{Tet}(x) \wedge \text{Medium}(x)))$
7. $\forall x\ ((\text{Tet}(x) \wedge \text{Small}(x)) \leftrightarrow x = b)$
8. $\exists y\ (y \neq e \wedge \text{SameRow}(y, e))$

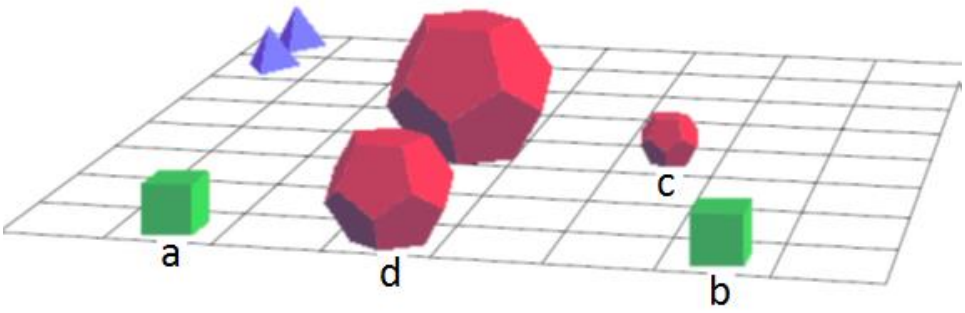


5. Draw a world in which Aristotle's sentences, given below, are all true. [5 marks]

1. $\exists x\ (\text{Tet}(x) \wedge \text{Large}(x))$
2. $\exists x\ (\text{Tet}(x) \wedge \text{Medium}(x))$
3. $\exists x\ (\text{Cube}(x) \wedge \neg\text{Small}(x))$
4. $\exists y\ (\text{Dodec}(y) \wedge \neg\text{Large}(y))$
5. $\forall x\ (\text{Cube}(x) \rightarrow \text{Medium}(x))$
6. $\forall x\ (\text{Dodec}(x) \rightarrow \text{Small}(x))$
7. $\forall x\ (\text{Tet}(x) \rightarrow \neg\text{Small}(x))$
8. $\forall y\ (\text{Cube}(y) \rightarrow \neg\text{Tet}(y))$

6. Translate the sentences below in FOL. (Every sentence is true in the world provided.) [1 mark each]

1. All the tetrahedra are small.
2. Every large thing is a dodecahedron.
3. Some dodecahedron is small.
4. Some dodecahedron is neither large nor small.
5. No tetrahedron is medium.



7. Translate the five sentences below into FOL. If correct, all five sentences will be true in the world from Question 6. [2 marks each]

1. Some small dodecs are in back of d.
2. Only dodecs are medium.
3. Every tetrahedron is both left of and the same size as c.
4. No large dodec is in the same row as b.
5. Every dodec that's in the same row as a is medium.