

### Answers to Problem Set 6

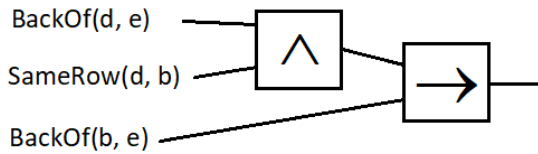
Total: 50 marks

1. [1 mark each]

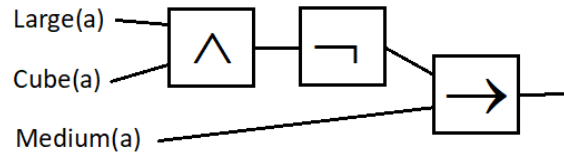
1.  $(\text{LeftOf}(a, d) \vee \text{RightOf}(a, d)) \rightarrow \text{Cube}(a)$
2.  $\text{LeftOf}(a, c) \rightarrow \text{RightOf}(e, a)$
3.  $\text{Tet}(e) \rightarrow (\text{RightOf}(e, b) \rightarrow \text{FrontOf}(e, b))$
4. If  $\underline{d}$  is both back of  $\underline{c}$  and in the same row as  $\underline{b}$ , then  $\underline{b}$  is back of  $\underline{c}$ .
5.  $\text{Cube}(d) \leftrightarrow \neg(\text{Tet}(d) \vee \text{Dodec}(d))$
6.  $\underline{a}$  is medium, unless it's a large cube.
7.  $\text{SameShape}(d, b) \rightarrow \text{SameSize}(d, b)$
8.  $\neg\text{Tet}(c) \rightarrow \text{Cube}(b)$
9. If both  $\underline{a}$  and  $\underline{d}$  are tetrahedra, then  $\underline{b}$  and  $\underline{e}$  are both cubes.
10. If  $\underline{b}$  and  $\underline{d}$  are the same size, then  $\underline{b}$  is large if and only if  $\underline{d}$  is.

Practice Exercise:

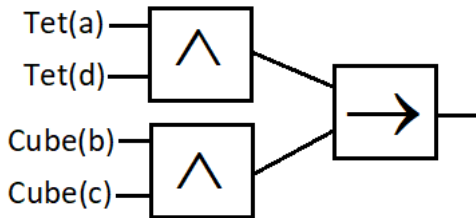
4.



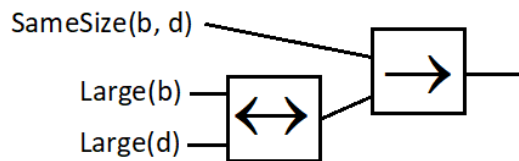
6.



9.



10.



2. (i) They are TT equivalent, from the absence of a counter-example row. [3 marks]

A	B	$\neg(A \rightarrow B)$		$A \wedge \neg B$	
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	F	T	F	T

(1)                      (2)

(ii) Not TT equivalent, from counter-example rows 6 and 8. [4 marks]

A	B	C	$(A \rightarrow B) \rightarrow C$	$A \rightarrow (B \rightarrow C)$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
*	F	T	T	F
F	F	T	T	T
*	F	F	T	T

3. (i) [5 marks]

1. $A \rightarrow B$	
2. $(A \wedge B) \rightarrow C$	
3. $(C \wedge D) \rightarrow E$	
4. $A \wedge D$	
5. $A$	✓ $\wedge$ Elim :4
6. $B$	✓ $\rightarrow$ Elim :1,5
7. $A \wedge B$	✓ $\wedge$ Intro :5,6
8. $C$	✓ $\rightarrow$ Elim :2,7
9. $D$	✓ $\wedge$ Elim :4
10. $C \wedge D$	✓ $\wedge$ Intro :8,9
11. $E$	✓ $\rightarrow$ Elim :3,10
12. $(A \wedge D) \rightarrow E$	✓ $\rightarrow$ Intro :4-11

(ii) [5 marks] **Not TT con.** (N.B. The row shown below is the *only* T || F row.)

<b>T</b>	$B \rightarrow C$	$A$	$B$	$C$	$B \rightarrow C$	$(A \vee B) \rightarrow C$
	----					
<b>F</b>	$(A \vee B) \rightarrow C$	<b>T</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>

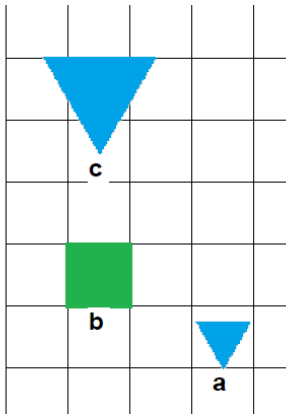
4. (i) [8 marks]

1. $H \rightarrow (E \wedge D)$	
2. $(E \vee P) \rightarrow R$	
3. $M \rightarrow \neg R$	
<hr/>	
4. $H$	
<hr/>	
5. $E \wedge D$	✓ $\rightarrow$ Elim :1,4
6. $E$	✓ $\wedge$ Elim :5
7. $E \vee P$	✓ $\vee$ Intro :6
8. $R$	✓ $\rightarrow$ Elim :2,7
<hr/>	
9. $M$	
<hr/>	
10. $\neg R$	✓ $\rightarrow$ Elim :3,9
11. $\perp$	✓ $\perp$ Intro :8,10
12. $\neg M$	✓ $\neg$ Intro :9-11
13. $H \rightarrow \neg M$	✓ $\rightarrow$ Intro :4-12

(ii) [8 marks]

1. $(B \vee G) \wedge \neg(B \wedge G)$	
2. $B \rightarrow \neg H$	
<hr/>	
3. $B \vee G$	✓ $\wedge$ Elim :1
4. $\neg(B \wedge G)$	✓ $\wedge$ Elim :1
<hr/>	
5. $G \vee H$	
<hr/>	
6. $B$	
<hr/>	
7. $\neg H$	✓ $\rightarrow$ Elim :2,6
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8. $G$	
<hr/>	
9. $B \wedge G$	✓ $\wedge$ Intro :6,8
10. $\perp$	✓ $\perp$ Intro :4,9
<hr/>	
11. $H$	
<hr/>	
12. $\perp$	✓ $\perp$ Intro :7,11
13. $\perp$	✓ $\vee$ Elim :5,8-10,11-12
14. $\neg B$	✓ $\neg$ Intro :6-13
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15. $\neg B$	
<hr/>	
16. $B$	
<hr/>	
17. $\perp$	✓ $\perp$ Intro :15,16
18. $G \vee H$	✓ $\perp$ Elim :17
<hr/>	
19. $G$	
<hr/>	
20. $G \vee H$	✓ $\vee$ Intro :19
21. $G \vee H$	✓ $\vee$ Elim :16-18,19-20,3
22. $(G \vee H) \leftrightarrow \neg B$	✓ $\leftrightarrow$ Intro :5-14,15-21

5. [7 marks]



- |   |  |
|---|--|
| T | 1. $\text{Cube}(b) \rightarrow (\text{FrontOf}(b, c) \wedge \text{SameCol}(b, c))$ |
| T | 2. $\text{Dodec}(a) \rightarrow a = b$   |
| T | 3. $\neg(\text{BackCf}(b, c) \vee \neg\text{SameShape}(a, c))$                     |
| F | 4. $\text{Cube}(b) \rightarrow \text{SameShape}(a, b)$                             |

In your world you need:

Cube(b)  
Tet(a)  
Tet(c)

FrontOf(b, c)  
SameCol(b, c)

(a can be anywhere)