Philosophy 1102
Introduction to Logic

Instructor: Richard Johns

Problem Set 6

Submit answers to the following questions in class on Thursday, February 15, 2024.
Please don't try to cram your answers onto this sheet!

1. Translate the following English sentences into FOL, and the FOL sentences into clear, simple English. (Note that your translations, if correct, will all be true in Bolzano's World below.) [1 mark each]

1. If <u>a</u> is to the left of or right of <u>d</u> then <u>a</u> is a cube.
2. <u>e</u> is to the right of <u>a</u>, provided <u>a</u> is left of <u>c</u>.
3. If <u>e</u> is a tetrahedron, then it's to the right of <u>b</u> only if it is also in front of <u>b</u>.
4. (BackOf(d, c) ∧ SameRow(d, b)) → BackOf(b, c)
5. <u>d</u> is a cube if and only if it is neither a tetrahedron nor a dodecahedron.
6. ¬(Large(a) ∧ Cube(a)) → Medium(a)   [Use the word 'unless' in your English sentence.]
7. <u>d</u> is the same shape as <u>b</u> only if they are the same size.
8. <u>b</u> is a cube unless <u>c</u> is a tetrahedron.
9. (Tet(a) ∧ Tet(d)) → (Cube(b) ∧ Cube(e))
10. SameSize(b, d) → (Large(b) ↔ Large(d))
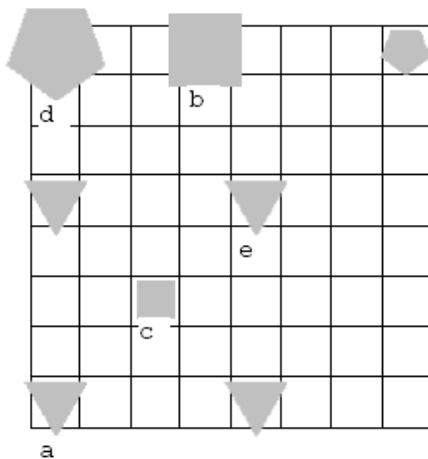
Practice Exercise [0 marks]

[Do these if you think they're useful.]

Draw circuit diagrams for sentences 4, 6, 9 and 10 above.

Solutions will be provided.

2. For each of the following pairs of sentences, use a truth table to determine whether or not they are TT equivalent. For a full answer, do all steps (a)-(d):

(a) Complete the *full* truth table.
(b) Write your verdict (either "TT equivalent" or "not TT equivalent").
(c) Write either "from a counter-example row", or "from the absence of a counter-example row", to explain how you obtained your verdict.
(d) If you wrote "from a counter-example row", then indicate *all* such rows with an asterisk (*).

(i) ¬ (A → B)   ⇔$_{TT}$?   A ∧ ¬ B   [3 marks]

(ii) (A → B) → C   ⇔$_{TT}$?   A → (B → C)   [4 marks]

3. For each argument below: if it's TT con then give a formal proof. If it's not TT con then give *one* counterexample (TT || F) row of the truth table. [5 marks each]

(i)

$A → B$
$(A ∧ B) → C$
$(C ∧ D) → E$
$\overline{(A ∧ D) → E}$

(ii)

$B → C$
$\overline{(A ∨ B) → C}$

4. Give formal proofs of the arguments below. [My proofs are 13 and 22 lines.]
[8 marks each]

(i)

$H → (E ∧ D)$
$(E ∨ P) → R$
$M → ¬R$
$\overline{H → ¬M}$

(ii)

$(B ∨ G) ∧ ¬(B ∧ G)$
$B → ¬H$
$\overline{(G ∨ H) ↔ ¬B}$

5. Draw a counter-example world to show that the argument below isn't a logical consequence. [7 marks]

Cube(b) → (FrontOf(a, b) ∧ SameCol(b, c))
Dodec(a) → a = b
¬(BackOf(b, c) ∨ ¬SameShape(a, c))
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Cube(b) → SameShape(a, b)