

## Answers to Problem Set 5

Total: 50 marks

1. [7 marks]

1. $A \vee B$	
2. $\neg B \vee C$	
3. $A$	
4. $A \vee C$	✓ $\vee$ Intro :3
5. $B$	
6. $\neg B$	
7. $\perp$	✓ $\perp$ Intro :5,6
8. $A \vee C$	✓ $\perp$ Elim :7
9. $C$	
10. $A \vee C$	✓ $\vee$ Intro :9
11. $A \vee C$	✓ $\vee$ Elim :6-8,9-10,2
12. $A \vee C$	✓ $\vee$ Elim :3-4,1,5-11

2. [7 marks]

1. $\neg A \wedge \neg B$	
2. $\neg A$	$\wedge$ Elim :1
3. $\neg B$	$\wedge$ Elim :1
4. $A \vee B$	
5. $A$	
6. $\perp$	$\perp$ Intro :5,2
7. $B$	
8. $\perp$	$\perp$ Intro :7,3
9. $\perp$	$\vee$ Elim :4,5-6,7-8
10. $\neg(A \vee B)$	$\neg$ Intro :4-9

3. [7 marks]

1. $A \vee B$	
2. $A \vee C$	
3. $A$	
4. $A \vee (B \wedge C)$	✓ ▾ $\vee$ Intro: 3
5. $B$	
6. $A$	
7. $A \vee (B \wedge C)$	✓ ▾ $\vee$ Intro: 6
8. $C$	
9. $B \wedge C$	✓ ▾ $\wedge$ Intro: 8,5
10. $A \vee (B \wedge C)$	✓ ▾ $\vee$ Intro: 9
11. $A \vee (B \wedge C)$	✓ ▾ $\vee$ Elim: 6-7,8-10,2
12. $A \vee (B \wedge C)$	✓ ▾ $\vee$ Elim: 3-4,5-11,1

4. [4 marks]

The argument is not a logical consequence, as the following world shows. (In this world  $\underline{b}$  is a small dodec, but it could instead be a *medium cube*.)



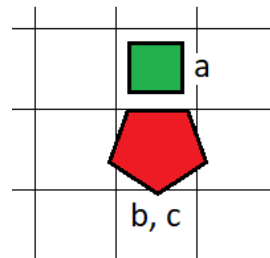
T	1. $\text{Dodec}(b) \vee \text{Cube}(b)$
T	2. $\text{Small}(b) \vee \text{Medium}(b)$
T	3. $\neg(\text{Small}(b) \wedge \text{Cube}(b))$
F	4. $\text{Medium}(b) \wedge \text{Dodec}(b)$

5. [7 marks]

1. $\neg\text{Small}(c) \vee \neg\text{Small}(a)$	
2. $\text{Small}(a)$	
3. $a = c$	
4. $\neg\text{Small}(c)$	
5. $\text{Small}(c)$	✓ = Elim :3,2
6. $\perp$	✓ $\perp$ Intro :4,5
7. $\neg\text{Small}(a)$	
8. $\perp$	✓ $\perp$ Intro :2,7
9. $\perp$	✓ $\vee$ Elim :1,4-6,7-8
10. $a \neq c$	✓ $\neg$ Intro :3-9

6. Not a logical consequence. [4 marks]

T	$\neg\text{Tet}(c) \vee \text{LeftOf}(a, b)$
T	$\text{Cube}(a) \wedge b = c$
T	$\neg\text{Dodec}(b) \vee (\text{Adjoins}(a, b) \wedge \text{BackOf}(a, b))$
F	$\neg\text{SameCol}(a, b) \vee \text{SameShape}(a, b)$



7. [7 marks]

1.	
2. $\neg(P \vee \neg P)$	
3. P	
4. $P \vee \neg P$	$\vee$ Intro :3
5. $\perp$	$\perp$ Intro :4,2
6. $\neg P$	$\neg$ Intro :3-5
7. $P \vee \neg P$	$\vee$ Intro :6
8. $\perp$	$\perp$ Intro :7,2
9. $\neg\neg(P \vee \neg P)$	$\neg$ Intro :2-8
10. $P \vee \neg P$	$\neg$ Elim :9

8. [7 marks]

1. $A \vee B$	
2. $\neg B$	
3. C	
4. $\neg(C \wedge A)$	
5. A	
6. $C \wedge A$	✓ $\wedge$ Intro :3,5
7. $\perp$	✓ $\perp$ Intro :4,6
8. B	
9. $\perp$	✓ $\perp$ Intro :2,8
10. $\perp$	✓ $\vee$ Elim :1,5-7,8-9
11. D	✓ $\perp$ Elim :10