

Answers to Problem Set 4

Total: 50 marks

1. [7 marks]

1.	$(A \vee B) \wedge \neg(C \wedge \neg D)$	
2.	$\neg C \wedge \neg(A \wedge B)$	
	<hr style="width: 100%;"/>	
3.	$(A \vee B)$	\wedge Elim: 1
4.	$\neg(A \wedge B)$	\wedge Elim: 2
5.	$(A \vee B) \wedge \neg(A \wedge B)$	\wedge Intro: 3, 4

2. [7 marks]

1	$(\neg A \vee C) \wedge B$	
2	C	
	<hr style="width: 100%;"/>	
3	B	\wedge Elim: 1
4	$A \vee B$	\vee Intro: 3
5	$(A \vee B) \wedge C$	\wedge Intro: 2, 4

3. [7 marks]

1.	$\neg(A \vee B)$	
	<hr style="width: 100%;"/>	
	2. ∇B	
	<hr style="width: 100%;"/>	
	3. $A \vee B$	✓ ∇ \vee Intro: 2
	4. \perp	✓ ∇ \perp Intro: 1,3
	<hr style="width: 100%;"/>	
5.	$\neg B$	✓ ∇ \neg Intro: 2-4

4. [7 marks]

1. $\neg A$	
2. $\neg B$	
3. $B \vee A$	
4. B	
5. \perp	✓ \perp Intro :4,2
6. A	
7. \perp	✓ \perp Intro :6,1
8. \perp	✓ \vee Elim :6-7,4-5,3
9. $\neg(B \vee A)$	✓ \neg Intro :3-8

5. [7 marks]

1. $(A \wedge B) \vee C$	
2. $\nabla A \wedge B$	
3. B	✓ $\nabla \wedge$ Elim: 2
4. $C \vee B$	✓ $\nabla \vee$ Intro: 3
5. ∇C	
6. $C \vee B$	✓ $\nabla \vee$ Intro: 5
7. $C \vee B$	✓ $\nabla \vee$ Elim: 1,2-4,5-6

6. The argument is invalid. [4 marks]

$\neg(A \vee B) \vee C$
$\neg A$
$\neg C$

A	B	C	$\neg(A \vee B) \vee C$	$\neg A$	$\neg C$
F	T/F	T	T	T	F

7. The argument is invalid. [4 marks]

$\text{Cube}(a) \vee \text{Cube}(b)$ $\neg(\text{Cube}(c) \wedge \text{Cube}(b))$ ----- $\neg\text{Cube}(c)$	$A \vee B$ $\neg(C \wedge B)$ ----- $\neg C$												
<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;">A</td> <td style="padding: 2px;">B</td> <td style="padding: 2px;">C</td> <td style="padding: 2px;">$A \vee B$</td> <td style="padding: 2px;">$\neg(C \wedge B)$</td> <td style="padding: 2px;">$\neg C$</td> </tr> <tr> <td style="padding: 2px; background-color: yellow;">T</td> <td style="padding: 2px; background-color: yellow;">F</td> <td style="padding: 2px; background-color: yellow;">T</td> <td style="padding: 2px;">T T F</td> <td style="padding: 2px;">T T F F</td> <td style="padding: 2px;">F T</td> </tr> </table>	A	B	C	$A \vee B$	$\neg(C \wedge B)$	$\neg C$	T	F	T	T T F	T T F F	F T	(I.e. <u>a</u> and <u>c</u> are cubes, but <u>b</u> isn't a cube.)
A	B	C	$A \vee B$	$\neg(C \wedge B)$	$\neg C$								
T	F	T	T T F	T T F F	F T								

8. [7 marks]

1. $A \vee (B \wedge C)$	
2. A	
3. $A \vee B$	✓ \vee Intro :2
4. $A \vee C$	✓ \vee Intro :2
5. $(A \vee B) \wedge (A \vee C)$	✓ \wedge Intro :3,4
6. $B \wedge C$	
7. B	✓ \wedge Elim :6
8. $A \vee B$	✓ \vee Intro :7
9. C	✓ \wedge Elim :6
10. $A \vee C$	✓ \vee Intro :9
11. $(A \vee B) \wedge (A \vee C)$	✓ \wedge Intro :8,10
12. $(A \vee B) \wedge (A \vee C)$	✓ \vee Elim :2-5,6-11,1