Philosophy 1102

Instructor: Richard Johns

Problem Set 3

Hand in answers to the following questions in class on Thursday, January 25.

[Total: 50 marks]

1. Translate the following sentences into FOL, using the Tarski's World predicates (see p. 22), the identity predicate =, and the sentential operators ∧, ∨ and ¬. If correct, your answers will all be true in the world below. [2 marks each, 16 total]

1. d and e are both the same size as f.
2. It is not the case that b is in the same row as c.
3. d is both in back of a and larger than it.
4. Neither d nor b is a tetrahedron.
5. c and f are not both left of b.
6. a is either left of both d and f or smaller than both of them.
7. Neither e nor f is smaller than either c or b.
8. Neither d nor f is both the same shape as, and in the same row as, c.
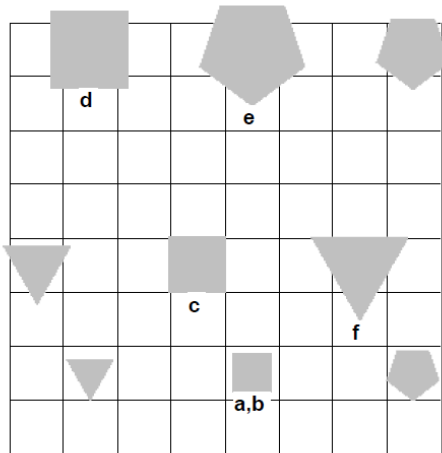
1.

2.

3.

4.

5.

6.

7.

8.

2. For each of the following sentences, use a truth table to determine whether or not it is TT true (i.e. a tautology, or TT necessary). For a full answer, do all steps (a)-(d):

(a) Complete the full truth table.
(b) Write your verdict (either "TT true" or "not TT true").
(c) Write either "from a counter-example row", or "from the whole table", according to whether or not there are one or more counter-example rows in the table.
(d) If you write "from a counter-example row", then indicate *all* such rows with an asterisk (*).

(i) $(\neg A \lor B) \lor (A \land \neg B)$ [4 marks]
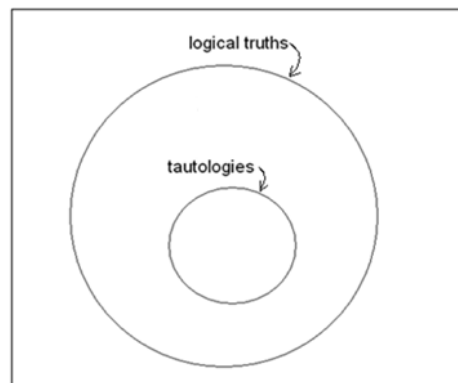
(ii) $(A \land \neg B) \lor (\neg A \land B)$ [4 marks]

3. For each of the following sentences, indicate whether or not it is a logical truth, and whether or not it is a tautology. (One has been done for you as an example.)
[1 mark each, 5 total]

|		logical truth?	tautology?
E.g.	1 + 1 = 2	Yes	No
(i)	Dodec(b) $\lor \neg$ Dodec(b)		
(ii)	$\neg$ (Cube(a) $\lor$ Larger(a, b))		
(iii)	$\neg$ Larger(a, b) $\lor \neg$ SameSize(b, a)		
(iv)	$\neg$ (LeftOf(a, b) $\land$ a = b)		
(v)	$\neg$ (Smaller(a, b) $\land \neg$ Smaller(a, b))		

[*Hint*: Remember that every tautology is a logical truth!]

4. For each of the following sentences, indicate whether or not it is logically possible, and whether or not it is 'TW possible', i.e. possible in Tarski's world. (One has been done for you as an example.) [1 mark each, 5 total]

		Logically possible?	**TW possible?**
E.g.	Larger(c, e) ∧ Large(e)	Yes	No
(i)	Larger(b, a) ∧ Smaller(b, a)		
(ii)	SameSize(a, b) ∧ Small(a) ∧ Large(b)		
(iii)	Tet(a) ∧ BackOf(a, b)		
(iv)	Medium(c) ∧ Larger(a, c) ∧ ¬Large(a)		
(v)	¬ Cube(a) ∧ ¬Tet(a) ∧ ¬Dodec(a))		

[Hint: Remember that every sentence that is TW possible is also logically possible!]

logically possible

TW possible

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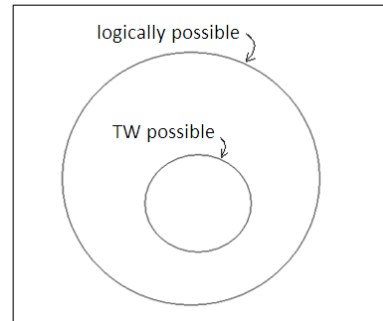
5. (i) Write each of the two arguments below in 'Boolean goggles' form, using the letters A, B, C etc. as needed, in the same order that they first appear in each argument. [2 marks total]

(a)

$\neg(\text{Larger}(a, b) \wedge \text{Tet}(c))$
$\text{Larger}(a, b)$

$\neg\text{Tet}(c)$

(b)

$\text{Larger}(a, b) \vee \text{Cube}(c)$
$a = b$

$\neg\text{Tet}(c)$

(ii) Say whether or not each argument is a *logical* consequence, and whether or not it is a *tautological* (TT) consequence. [2 marks total]

(iii) Show that one of the arguments above is not TT con by filling in *one row* of a truth table) [2 marks]

6. Use a truth table to determine whether or not each of the following pairs of sentences are tautologically (TT) equivalent.

(i) $A \wedge (B \vee C)$ $(A \wedge B) \vee (A \wedge C)$ [5 marks]

(ii) $(P \wedge Q) \vee R$ $P \wedge (Q \vee R)$ [5 marks]

For a satisfactory answer do the same four things (a) – (d) from Qu. 2 above, except that the verdict in each case will be either 'TT equivalent' or 'Not TT equivalent'.

A	B	C	$A \wedge (B \vee C)$	$(A \wedge B) \vee (A \wedge C)$

P	Q	R	$(P \wedge Q) \vee R$	$P \wedge (Q \vee R)$

The sentences are _____

From _____.

The sentences are _____

From _____.

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