Philosophy 1102
Introduction to Logic

Instructor: Richard Johns

Problem Set 2

Hand in answers to the following questions at the start of class on Thursday, Jan. 18.

1. If any of the following arguments is valid, provide a *proof outline* (see example).

    [Your proof outline should insert two sentences as *steps in between* the premises and conclusion, as shown in the example below. Also write down the line numbers of the earlier statements used – **a maximum of two** – next to each step.]

    If it isn't valid then draw a *counterexample world*. [4 marks each]

	argument	Proof outline
Example:	a = b BackOf(b, c) SameRow(c, d) ------- FrontOf(d, a)	1. a = b 2. BackOf(b, c) 3. SameRow(c, d) ------- 4. BackOf(a, c) (1, 2) 5. BackOf(a, d) (3, 4) 6. FrontOf(d, a) (5)

(i)   Adjoins(a, b)   (ii)   Tet(b) ∨ Cube(c)   (iii)   SameSize(b, c)
        SameRow(c, b)   Tet(a)   Tet(c) ∧ Tet(b)
        LeftOf(a, c)   ¬ SameShape(a, b)   SameRow(b, c)
        ---------   -----------   ------
        SameRow(a, c)   ¬ Dodec(c)   b = c
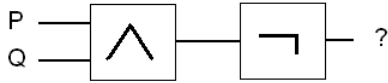
2. Show that the following argument is valid by providing a *formal* proof. (Fill in the lines 4, 5 and 6, citing rules of inference, and fill in the rule for line 7.) [5 marks]

    1 | RightOf(a, c)
    2 | a = b
    3 | d = c
    4 |
    5 |
    6 |
    7 | RightOf(b, d)

3. Translate each 'circuit diagram' below into a FOL sentence, and translate each FOL sentence into a 'circuit diagram'. [1 mark each, 6 total]
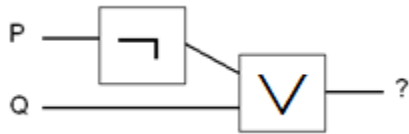
Also write down the *type* of each sentence, i.e. write "conjunction sentence', 'negation sentence', or 'disjunction sentence' as appropriate. (It's always the last operator applied.) [1 mark each, 6 total]
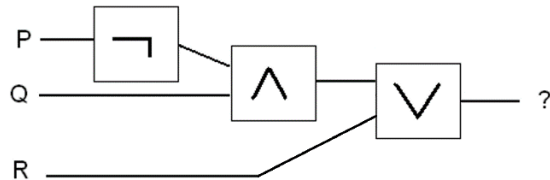
(i)

P
Q
$\wedge$ — $\neg$ — ?
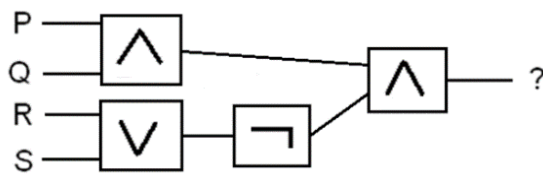
Sentence type _____

(ii)

P — $\neg$
Q — $\vee$ — ?

Sentence type _____

(iii)

P — $\neg$
Q — $\wedge$ — $\vee$ — ?
R

Sentence type _____

(iv)

P — $\wedge$
Q
R — $\vee$ — $\neg$ — $\wedge$ — ?
S

Sentence type _____

(v) $\neg(P \vee Q) \wedge R$

Sentence type _____

(vi) $\neg(((P \vee Q) \wedge R) \vee S)$

Sentence type _____

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4. For each of the following sentences, write 'T' or 'F' next to it, according to whether it is true or false in the world shown below. [Total 10 marks]

(Please do not use the program Tarski's World to generate your answers – figure it out yourself, perhaps using a circuit diagram. You may, however, use the program to *check* your answers.)

1. d ≠ e   (i.e. ¬d = e, the negation of d = e)
_____
2. Dodec(c) ∨ Dodec(a)
_____
3. Cube(d) ∧ Cube(e)
_____
4. ¬(Cube(d) ∧ Cube(f))
_____
5. ¬(Cube(a) ∨ Cube(f))
_____
6. ¬(Small(a) ∧ ¬Small(f))
_____
7. Dodec(f) ∨ ¬Medium(a) ∨ Tet(e)
_____
8. ¬(Large(c) ∧ ¬Medium(a)) ∨ ¬ ¬Small(f)
_____
9. (SameSize(a, f) ∨ SameSize(d, e) ∨ Larger(c, f)) ∧ ¬Larger(f, a)
_____
10. ¬ ¬ ¬ ¬Medium(d)
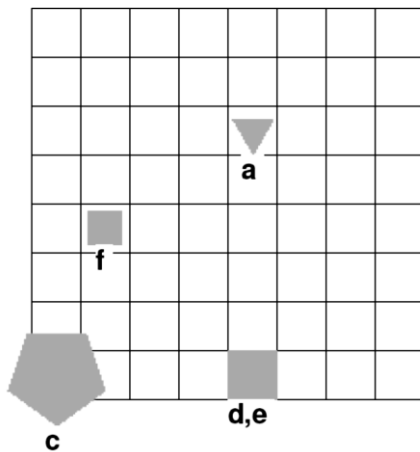
5. Draw a world in which *all* the sentences from Qu. 4 are true. (A couple of modifications to the world shown above will do it.) [5 marks for world]

6. Show that the following arguments are invalid, by drawing a counterexample world for each one. [3 marks each]

(i) ¬(Large(a) ∧ Large(b))
     _____
     ¬Large(a) ∧ Large(b)

(ii) Tet(a) ∨ (Tet(b) ∧ Tet(c))
      _____
      (Tet(a) ∨ Tet(b)) ∧ Tet(c)