# Boolean Goggles

Revealing the *form* of an FOL sentence

# Boolean (TT) Goggles


(Unfortunately we are out of stock at present)

# How they work

- Boolean goggles are used to reveal the Boolean *form* (= Boolean structure, pattern) of FOL sentences.
- The Boolean form of the sentence is clearly visible through the goggles
  - but everything else becomes fuzzy and illegible.

# How they work

- The Boolean sentential operators $\wedge$, $\vee$, $\neg$, $\rightarrow$ and $\leftrightarrow$ are clearly visible through the goggles.
- Brackets '(' and ')' are also visible, *if* they're created by the Boolean operators.
- A special sentence $\bot$ is visible.
- All other atomic sentences become fuzzy and unreadable
  – but you can see if two atomic sentences are *exactly* the same.

# E.g.

Tet(a) $\land$ Tet(b)

- Through Boolean goggles you see:


Which we write as: $P \land Q$

(That's the 'Boolean form' of the sentence)

$\text{Tet}(a) \lor \text{Dodec}(a)$

$\neg\text{Tet}(a)$

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$\text{Dodec}(a)$

- Through Boolean goggles you see:

$\text{Tet}(a) \lor \text{Dodec}(a)$

$\neg\ \text{Tet}(a)$

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$\text{Dodec}(a)$

$A \lor B$

$\neg A$

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$B$

- E.g. $(\text{Tet}(a) \rightarrow \neg\text{Cube}(a)) \wedge \text{Dodec}(c)$

- Through Boolean goggles you see:

  ( [illegible] $\rightarrow \neg$ [illegible] ) $\wedge$ [illegible]

  Which we write as: $(P \rightarrow \neg Q) \wedge R$

Cube(a)

a = b

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Cube(b)

Cube(a)

a = b

---------

Cube(b)

Through B.G.:

P

Q

-----

R

Cube(a) ∨ ¬Small(a)

Small(a) ∨ SameRow(a, c)

FrontOf(a, c) ∨ ¬Large(a)

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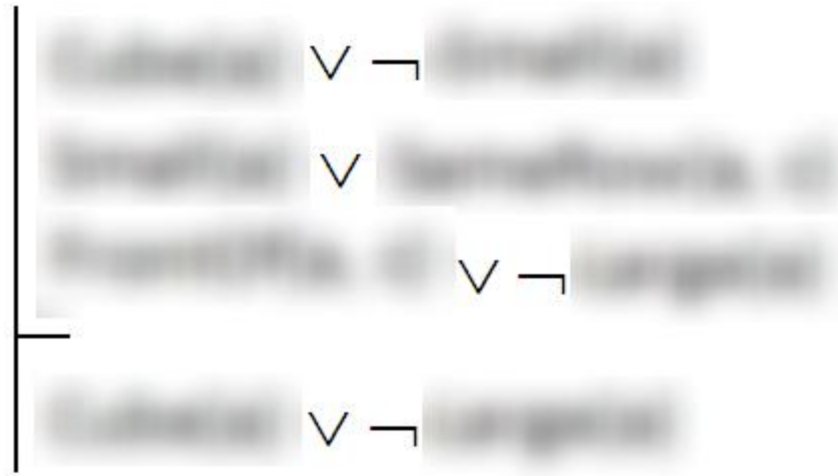
Cube(a) ∨ ¬Large(a)



A ∨ ¬B

B ∨ C

D ∨ ¬E

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A ∨ ¬E

# Validity and TT goggles

- Suppose you're looking at an argument with TT goggles on, and you see:

$P \vee Q$

$\neg P$

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$Q$

  – Can you tell if it's a *valid* argument?

- **Yes**. The meanings of the sentences P and Q are *irrelevant* here.
- There are only 4 possible combinations of truth values for P and Q, namely TT, TF, FT and FF.
- None of these makes the premises true and the conclusion false.
  - (Check with a truth table.)
- Hence *there is no possible world in which the premises are true and the conclusion false*.
  - Hence the argument is valid.

# Truth table for the argument

| P | Q | $P \vee Q$ | $\neg P$ | Q |
|---|---|---|---|---|
| T | T | | | |
| T | F | | | |
| F | T | | | |
| F | F | | | |

# Truth table for the argument

| $P$ | $Q$ | $P \vee Q$ | $\neg P$ | $Q$ |
|---|---|---|---|---|
| T | T | T | | |
| T | F | T | | |
| F | T | T | | |
| F | F | F | | |

# Truth table for the argument

| $P$ | $Q$ | $P \vee Q$ | $\neg P$ | $Q$ |
|---|---|---|---|---|
| T | T | T | F | |
| T | F | T | F | |
| F | T | T | T | |
| F | F | F | T | |

# Truth table for the argument

| P | Q | $P \vee Q$ | $\neg P$ | Q |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | T | F | F |
| F | T | T | T | T |
| F | F | F | T | F |

# Truth table for the argument

| P | Q | $P \vee Q$ | $\neg P$ | Q |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | T | F | F |
| F | T | T | T | T |
| F | F | F | T | F |

Is there a counter-example (TT|F) world?

# Truth table for the argument

| P | Q | $P \vee Q$ | $\neg P$ | Q |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | T | F | F |
| F | T | T | T | T |
| F | F | F | T | F |

Is there a counter-example (TT|F) world?

- No. (So, the argument is valid)

- Hence *we can test for logical consequence with a truth table*.

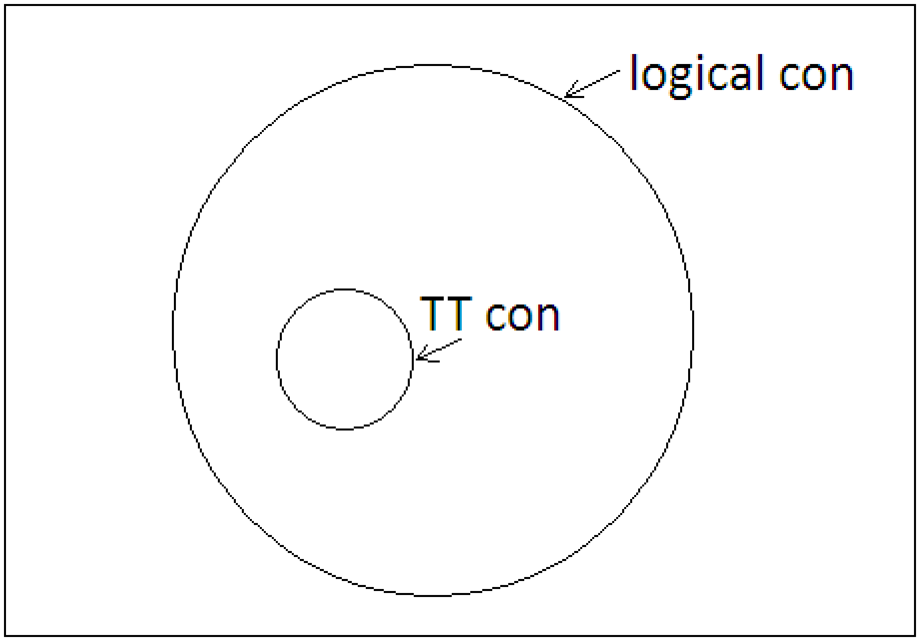
- If a truth table has ***no TT|F row*** (no row with true premises and a false conclusion) then it must be a logical consequence.

- In fact, this is a special kind of logical consequence, called tautological (or TT) consequence.

An argument that isn't TT consequence can still be a logical consequence.

TT consequence $\Rightarrow$ logical consequence

*But not vice-versa!*



- Is this argument a logical consequence?

P

Q

----

R

Cube(a)

a = b

---------

Cube(b)

Actually *yes*.

But it isn't a TT consequence.  For a TT con is a consequence that you can see through the TT goggles.

The actual argument is:

Cube(a)

a = b

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Cube(b)

# Which arguments are TT con?

$$\begin{array}{|l} \neg(\text{Small}(a) \lor \text{Dodec}(a)) \\ \hline \neg\text{Dodec}(a) \end{array}$$

$$\begin{array}{|l} \neg(\text{Cube}(a) \land \text{Large}(a)) \\ \text{Large}(a) \\ \hline \neg\text{Cube}(a) \end{array}$$

$$\begin{array}{|l} \text{Small}(a) \lor \text{Cube}(a) \\ \text{Small}(a) \\ \hline \neg\text{Cube}(a) \end{array}$$

$$\begin{array}{|l} \text{Larger}(a, b) \\ \text{Larger}(b, c) \\ \hline \text{Larger}(a, c) \end{array}$$

# Which arguments are TT con?

$$
\begin{array}{|l}
\neg(\mathrm{Small}(a) \lor \mathrm{Dodec}(a)) \\
\hline
\neg\mathrm{Dodec}(a)
\end{array}
$$

Yes, TT con

$$
\begin{array}{|l}
\neg(\mathrm{Cube}(a) \land \mathrm{Large}(a)) \\
\mathrm{Large}(a) \\
\hline
\neg\mathrm{Cube}(a)
\end{array}
$$

Yes, TT con

$$
\begin{array}{|l}
\mathrm{Small}(a) \lor \mathrm{Cube}(a) \\
\mathrm{Small}(a) \\
\hline
\neg\mathrm{Cube}(a)
\end{array}
$$

No, not TT con

(Not logical con either)

$$
\begin{array}{|l}
\mathrm{Larger}(a, b) \\
\mathrm{Larger}(b, c) \\
\hline
\mathrm{Larger}(a, c)
\end{array}
$$

No, not TT con

(but it is logical con)

# Truth table for not TT con?

$$\begin{array}{|l} \text{Small}(a) \lor \text{Cube}(a) \\ \text{Small}(a) \\ \hline \neg\text{Cube}(a) \end{array}$$

Boolean Goggles:

$$\begin{array}{|l} A \lor B \\ A \\ \hline \neg B \end{array}$$

| | A | B | $A \lor B$ | A | $\neg B$ |
|---|---|---|---|---|---|
| * | T | T | T | T | F |
| | T | F | T | T | T |
| | F | T | T | F | F |
| | F | F | F | F | T |

Is there a TT | F row?

Yes!

# TT true (i.e. tautology)

- Consider the sentence $\text{Tet(a)} \vee \neg\text{Tet(a)}$.

  It's a logical truth, since it's true in all possible worlds.

  Through the TT goggles it becomes:

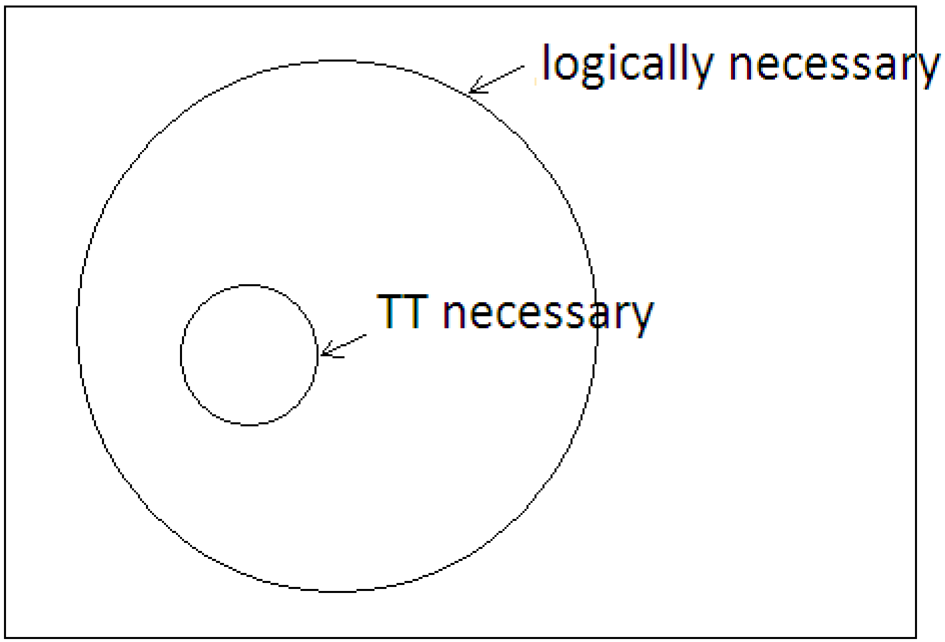
$$\text{Tet(a)} \vee \neg\text{Tet(a)} \qquad P \vee \neg P$$

It's true in every row of the TT, so it's TT true, i.e. **TT necessary**, i.e. a *tautology*.

- We know that 1+1=2.  A world where was false would be absurd.
- Hence it is a logical truth (a.k.a. logical necessity)
- But through the TT goggles we see only:

P

And so it isn’t **TT necessary**.


logically necessary
TT necessary