What's wrong (and right) with ⊥Elim?

AKA the Principle of Explosion, *Ex falso quodlibet*

What is \perp ?

- ⊥ is a sentence of FOL, that should be considered a short way of writing Q ∧ ¬Q, where Q is some arbitrary sentence.
- Note that ⊥ is therefore logically (and TT) impossible. It is false in every possible world, and false in every row of the truth table.
- Since ⊥ is short for Q ∧ ¬Q, and Q ∧ ¬Q is visible through the Boolean goggles, ⊥ is also visible through the goggles.

What is \perp Elim?

$\perp \operatorname{Elim}$



After introducing \perp , you can write any sentence you like underneath.

But you have to stay inside the subproof!

Criticisms of \perp Elim

- 1. The sentence \perp doesn't even have a meaning!
- Rules of inference are supposed to provide rational guidance. But ⊥Elim tells us to do something irrational and silly.
- 3. If we prove ⊥ in a subproof, then that subproof represents a "state of belief" that is irrational and impossible to be in. It's sometimes called "epistemic hell". So we need to get out, ASAP!

1. Does \perp have a meaning?

- On one account, the meaning of a sentence is a *proposition*, which in turn is "the content of a possible rational belief".
- Can a rational person believe Q ∧ ¬Q? It hardly seems so. But in that case the sentence has no meaning, and can't be the premise of an inference.
- Similarly: is "square circle" a mathematical property? No. There's no such thing in mathematics.
 - It would be silly to prove that every square circle has 4 sides, and also 10 sides, 1 side, 272636273 sides, etc.

Meaning = truth conditions?

- On the view that the meaning of a sentence is its truth conditions (i.e. the set of possible worlds where it's true) \perp *does* have a meaning, namely the empty set \emptyset .
- Then, according to the standard definition, every sentence P is a *logical* consequence of ⊥.
 - Since there's no possible world where \perp is true and P is false.
- Also, every sentence is a *TT* consequence of \perp .
 - Since there's no row of the truth table where \perp is true and P is false.

2. Rational Guidance

- We humans are frail, and often stray from the path of rationality.
 - The rules of inference are supposed to be like signposts that keep us on the path, if we obey them.



Except for \perp Elim, it seems!

3. Get out of epistemic hell!



You can end a subproof at any time you like, and get out of it.

Surely, after introducing ⊥, you should leave right away!

Response to 2 and 3

- The signpost picture is misleading, as it suggests that ⊥Elim is leading us off the path of rationality.
- But ⊥Elim can only be used when we already believe ⊥. In other words, we are already off the path of reason.
 - We're already in epistemic hell!
- Once we believe ⊥, nothing we do can make things any worse in that subproof. Of course we have to get out, but there's no harm in adding an extra belief or two before we leave.

Other reasons why $\perp Elim$ is OK

- A. \perp Elim is *unavoidable*
- B. \perp Elim is (occasionally) *useful*
- C. \perp Elim is (mostly) *harmless*

A. \perp Elim is unavoidable

 At least if we want Elim, Intro and D.S. to be valid in our system.

J.S. PvQ 70 11 17 5 P P.S: 3,4

B. \perp Elim is (occasionally) useful



C. \perp Elim is (mostly) harmless

• As long as you stay in the subproof.

1.
$$Tef(c)$$

2. $Tef(c)$
3. L
 $(whela) / LTuto: 1, 2$
4. $(whela) / Ltlim: 3$