Semantics of FOL

What does it all mean?

"Extensional" semantics

First the *domain* is defined – the set of things the language is talking about. The meanings of parts of FOL are defined *for this domain*.

part of FOL	meaning
name (individual constant)	Object (member of the domain)
One-place predicate P(x)	set of objects {x: P(x)}
Two-place predicate P(x, y)	set of ordered pairs { <x, y="">: P(x, y)}</x,>
Three-place predicate P(x, y, z)	set of ordered triples { <x, y,="" z="">: P(x, y, z)}</x,>
	(etc.)
Wff (non-sentence)	set of objects, or pairs, or triples (etc.)
Sentence	truth value (in this world)

<u>a</u> satisfies P(x) iff $\underline{a} \in \{x: P(x)\}$ (etc.)

"Extensional" semantics

- The semantics is considered 'extensional' because it defines meanings rather crudely.
- E.g. the names 'Superman' and 'Clark Kent' have the same meaning, even though their 'intensions' differ.
- In some worlds, 'Cube(x)' and 'Small(x)' will have the same meaning.

"Extensional" semantics

• In this world, {x: Cube(x)} = {x: Small(x)}



Boolean operators

E.g. **Conjunction wffs**: The meaning of (Cube(x) \land Large(x)) is the set: {x: Cube(x) \land Large(x)}, i.e. {x: Cube(x)} \cap {x: Large(x)}.

Negation wffs: The meaning of \neg Cube(x) is the set: {x: \neg Cube(x)}.

Quantifiers

- ∀x Cube(x) is T iff Cube(x) is satisfied by every object in the domain.
- ∃x Cube(x) is T iff Cube(x) is satisfied by at least one object in the domain.