Elementary Proof in F+ that 1+1 = 2

1.	$\exists x\, (F(x) \wedge \forall y\, (F(y) \rightarrow y = x))$	
2.	$\exists x\, (G(x) \wedge \forall y\, (G(y) \rightarrow y = x))$	
3.	$\neg \exists x\, (F(x) \wedge G(x))$	
4.	$\boxed{a}$ $F(a) \wedge \forall y\, (F(y) \rightarrow y = a)$	
5.	$F(a)$	$\wedge$ Elim: 4
6.	$\forall y\, (F(y) \rightarrow y = a)$	$\wedge$ Elim: 4
7.	$\boxed{b}$ $G(b) \wedge \forall y\, (G(y) \rightarrow y=b)$	
8.	$G(b)$	$\wedge$ Elim: 7
9.	$\forall y\, (G(y) \rightarrow y=b)$	$\wedge$ Elim: 7
10.	$\boxed{c}$ $F(c) \vee G(c)$	
11.	$F(c)$	
12.	$F(c) \rightarrow c=a$	$\forall$ Elim: 6
13.	$c=a$	$\rightarrow$ Elim: 11,12
14.	$c=a \vee c=b$	$\vee$ Intro: 13
15.	$G(c)$	
16.	$G(c) \rightarrow c=b$	$\forall$ Elim: 9
17.	$c=b$	$\rightarrow$ Elim: 15,16
18.	$c=a \vee c=b$	$\vee$ Intro: 17
19.	$c=a \vee c=b$	$\vee$ Elim: 15-18,11-14,
20.	$\forall z\, ((F(z) \vee G(z)) \rightarrow (z = a \vee z = b))$	$\forall$ Intro: 10-19
21.	$a=b$	
22.	$F(b)$	$=$ Elim: 21,5
23.	$F(b) \wedge G(b)$	$\wedge$ Intro: 22,8
24.	$\exists x\, (F(x) \wedge G(x))$	$\exists$ Intro: 23
25.	$\perp$	$\perp$ Intro: 24,3
26.	$a \neq b$	$\neg$ Intro: 21-25
27.	$F(b) \vee G(b)$	$\vee$ Intro: 8
28.	$F(a) \vee G(a)$	$\vee$ Intro: 5
29.	$(F(a) \vee G(a)) \wedge (F(b) \vee G(b)) \wedge a \neq b \wedge \forall z\, ((F(z) \vee G(z)) \rightarrow (z = a \vee z = b))$	$\wedge$ Intro: 20,26,27,28
30.	$\exists x\, \exists y\, ((F(x) \vee G(x)) \wedge (F(y) \vee G(y)) \wedge x \neq y \wedge \forall z\, ((F(z) \vee G(z)) \rightarrow (z = x \vee z = y)))$	$\exists$ Intro: 29
31.	$\exists x\, \exists y\, ((F(x) \vee G(x)) \wedge (F(y) \vee G(y)) \wedge x \neq y \wedge \forall z\, ((F(z) \vee G(z)) \rightarrow (z = x \vee z = y)))$	$\exists$ Elim: 7-30,2
32.	$\exists x\, \exists y\, ((F(x) \vee G(x)) \wedge (F(y) \vee G(y)) \wedge x \neq y \wedge \forall z\, ((F(z) \vee G(z)) \rightarrow (z = x \vee z = y)))$	$\exists$ Elim: 1,4-31