LANGARA COLLEGE

Philosophy 1102

Introduction to Logic

ANSWERS TO THE THIRD FAKE FINAL EXAMINATION

SPECIAL INSTRUCTIONS:

Answer all questions. *Write your answers in the separate answer booklet*. If you get stuck on a question, go on to the next, and return to it later. Indeed, it is wise to read the whole paper before you start, and begin with the easiest questions. Including this cover page, and the sheet of rules of inference, this examination booklet should consist of five pages. Check that these are all present before the examination begins.

INSTRUCTOR: Richard Johns

1. Translate the following English sentences into FOL, and the FOL sentences into good, simple English. Take the set of all objects as your domain of discourse, and use only the predicates listed below. All the sentences are true in the world shown below. [3 marks each, total 24]

Cube(x)	Large(x)	Larger(x, y)	LeftOf(x, y)
Tet(x)	Medium(x)	Smaller(x, y)	RightOf(x, y)
Dodec(x)	Small(x)	SameSize(x, y)	FrontOf(x, y)
x = y	Between(x, y, z)	SameRow(x, y)	BackOf(x, y)
	Adjoins(x, y)	SameCol(x, y)	

(i) Every dodecahedron is large.

$\forall x(\text{Dodec}(x) \to \text{Large}(x))$

(ii) $\exists x(\text{Cube}(x) \land \forall y(\text{Tet}(y) \to \text{LeftOf}(y, x)))$

All the tets are left of some cube.

(iii) The small cube is in the same column as c.

$\exists x(\text{Small}(x) \land \text{Cube}(x) \land \forall y((\text{Small}(y) \land \text{Cube}(y)) \to x = y) \land \text{SameCol}(x, c))$

(iv) $\exists x \exists y\, (\text{Dodec}(x) \land \text{Dodec}(y) \land x \neq y \land \forall z\, (\text{Dodec}(z) \to (z = x \lor z = y)))$

There are exactly two (two and only two) dodecs.

(v) $\forall x((\text{Tet}(x) \land x \neq f) \to \text{LeftOf}(c, x))$

c is left of every tet except f

(vi) Every dodecahedron in the same row as a tetrahedron is larger than that tetrahedron.

$\forall x \forall y((\text{Dodec}(x) \land \text{Tet}(y) \land \text{SameRow}(x, y)) \to \text{Larger}(x, y))$ (Donkey!)

(vii) There are three or more tetrahedra.

$\exists x \exists y \exists z\, (\text{Tet}(x) \land \text{Tet}(y) \land \text{Tet}(z) \land x \neq y \land y \neq z \land x \neq z)$

(viii) c is the only dodecahedron that's to the right of a small tetrahedron.

$\forall x\, ((\text{Dodec}(x) \land \exists y(\text{Small}(y) \land \text{Tet}(y) \land \text{RightOf}(x, y))) \to x = c)$

2. Show that one of the arguments below is TT con by providing a proof in $\mathcal{F}^+$. Show that the other is *not* TT con by providing *one* suitable row of a truth table. [6 +5 marks]

(i)

| A → (B ∨ C)
| A → ¬B
| ¬(C ∨ D)
|—
| A → E

(ii)

| (A ∧ B) → C
|—
| (A → C) ∧ (B → C)

TT con	Not TT con

1. A → (B ∨ C)
2. A → ¬B
3. ¬(C ∨ D)
—
    4. A
    —
    5. B ∨ C   → Elim *:1,4*
    6. ¬B   → Elim *:2,4*
    7. C   D.S *:5,6*
    8. C ∨ D   ∨ Intro *:7*
    9. ⊥   ⊥ Intro *:8,3*
    10. E   ⊥ Elim *:9*
11. A → E   → Intro *:4-10*
	A B C
	
	T F F
	F T F
	
	(Just give *one* of these two rows.)

3. (i) Is the conclusion of the argument below a logical consequence of the premises? [2 marks]

$$\exists x(\text{Large}(x) \wedge \text{Cube}(x))$$
$$\exists z(\text{Large}(z) \wedge \text{Tet}(z))$$
$$\overline{\exists x \exists y(\text{Large}(x) \wedge \text{Large}(y) \wedge x \neq y)}$$

Yes! It's a logical consequence.

(ii) Re-write the argument, replacing all the non-logical predicates with nonsense predicates. I.e. show it as it appears through "first-order goggles". [4 marks]

$\exists x(P(x) \wedge Q(x))$

$\exists z(P(z) \wedge R(z))$

$\exists x \exists y(P(x) \wedge P(y) \wedge x \neq y)$

(iii) Show that the argument is not a first-order consequence (FO con) by giving a suitable interpretation of the nonsense predicates, and drawing a suitable world. [10 marks]

$\exists x(\text{Big}(x) \wedge \text{Dog}(x))$

$\exists z(\text{Big}(z) \wedge \text{Hairy}(z))$

$\exists x \exists y(\text{Big}(x) \wedge \text{Big}(y) \wedge x \neq y)$

(I'm assuming that the woman in the world is not big.)

(iv) Give a *formal proof* of the conclusion of the argument, using additional premises from the shape axioms listed below. [10 marks]

1. ¬∃x (Cube(x) ∧ Tet(x))	(A1)
2. ∃x (Large(x) ∧ Cube(x))	
3. ∃x (Large(x) ∧ Tet(x))	
4. [a] Large(a) ∧ Cube(a)	
5. Large(a)	✓ ∧ Elim *:4*
6. Cube(a)	✓ ∧ Elim *:4*
7. [b] Large(b) ∧ Tet(b)	
8. Large(b)	✓ ∧ Elim *:7*
9. Tet(b)	✓ ∧ Elim *:7*
10. a = b	
11. Cube(b)	✓ = Elim *:10,6*
12. Cube(b) ∧ Tet(b)	✓ ∧ Intro *:9,11*
13. ∃x (Cube(x) ∧ Tet(x))	✓ ∃ Intro *:12*
14. ⊥	✓ ⊥ Intro *:13,* A1
15. ¬a = b	✓ ¬ Intro *:10-14*
16. Large(a) ∧ Large(b) ∧ ¬a = b	✓ ∧ Intro *:5,8,15*
17. ∃x ∃y (Large(x) ∧ Large(y) ∧ ¬x = y)	✓ ∃ Intro *:16*
18. ∃x ∃y (Large(x) ∧ Large(y) ∧ ¬x = y)	✓ ∃ Elim *:3,7-17*
19. ∃x ∃y (Large(x) ∧ Large(y) ∧ ¬x = y)	✓ ∃ Elim *:2,4-18*

Shape Axioms

A1. ¬∃x (Cube(x) ∧ Tet(x))
A2. ¬∃x (Tet(x) ∧ Dodec(x))
A3. ¬∃x (Dodec(x) ∧ Cube(x))
A4. ∀x (Tet(x) ∨ Dodec(x) ∨ Cube(x))
A5. ∀x ∀y ((Cube(x) ∧ Cube(y)) → SameShape(x, y))
A6. ∀x ∀y ((Dodec(x) ∧ Dodec(y)) → SameShape(x, y))
A7. ∀x ∀y ((Tet(x) ∧ Tet(y)) → SameShape(x, y))
A8. ∀x ∀y ((SameShape(x, y) ∧ Cube(x)) → Cube(y))
A9. ∀x ∀y ((SameShape(x, y) ∧ Dodec(x)) → Dodec(y))
A10. ∀x ∀y ((SameShape(x, y) ∧ Tet(x)) → Tet(y))

4. For the following arguments, give a formal proof in $\mathcal{F}^{+}$. (The arguments are both FO con.) [8, 10 marks]

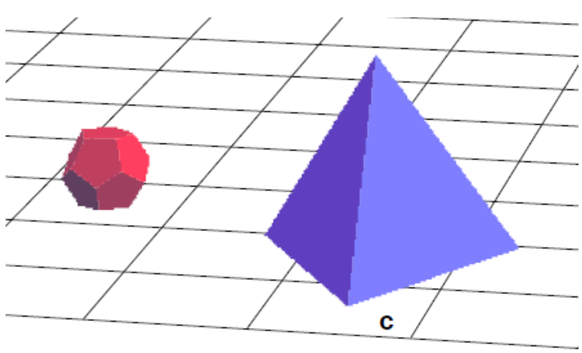
a.

1.	$\forall x\, (Cube(x) \to Large(x))$	
2.	$\forall y\, Cube(y)$	
3.	$\boxed{a}$	
4.	$Cube(a)$	✓ $\forall$ Elim :2
5.	$Cube(a) \to Large(a)$	✓ $\forall$ Elim :1
6.	$Large(a)$	✓ $\to$ Elim :4,5
7.	$\forall z\, Large(z)$	✓ $\forall$ Intro :3-6
8.	$\forall y\, Cube(y) \to \forall z\, Large(z)$	✓ $\to$ Intro :2-7

b.

1.	¬∃x (Cube(x) ∧ ¬Large(x))	
2.	∃y Cube(y)	
3.	∀x ∀y ((Cube(x) ∧ Cube(y)) → x = y)	
4.	[c] Cube(c)	
5.	¬Large(c)	
6.	Cube(c) ∧ ¬Large(c)	∧ Intro *:4,5*
7.	∃x (Cube(x) ∧ ¬Large(x))	∃ Intro *:6*
8.	⊥	⊥ Intro *:7,1*
9.	Large(c)	¬ Intro *:5-8*
10.	[a] Cube(a)	
11.	Cube(c) ∧ Cube(a)	∧ Intro *:4,10*
12.	(Cube(c) ∧ Cube(a)) → c = a	∀ Elim *:3*
13.	c = a	→ Elim *:11,12*
14.	∀y (Cube(y) → c = y)	∀ Intro *:10-13*
15.	Cube(c) ∧ ∀y (Cube(y) → c = y) ∧ Large(c)	∧ Intro *:4,9,14*
16.	∃x (Cube(x) ∧ ∀y (Cube(y) → x = y) ∧ Large(x))	∃ Intro *:15*
17.	∃x (Cube(x) ∧ ∀y (Cube(y) → x = y) ∧ Large(x))	∃ Elim *:4-16,2*

5. Show that the following sentences are not logically equivalent by drawing *one* counterexample world (in which they have different truth values). [5 marks]



F 1. ∀x (Dodec(x) → Small(c))

T 2. ∀x Dodec(x) → Small(c)

6. The sentence ***Every student is taking a course*** is ambiguous, and can be translated into FOL in two (non-equivalent) ways.

(i) Write down *both* translations, using the FOL predicates Student(x), Taking(x, y), (meaning '*x* is taking *y*') and Course(x). [3 + 3 marks]

$\forall x(\text{Student}(x) \rightarrow \exists y(\text{Course}(y) \wedge \text{Taking}(x, y)))$ ***weak***

$\exists y(\text{Course}(y) \wedge \forall x(\text{Student}(x) \rightarrow \text{Taking}(x, y)))$ ***strong***

(ii) One of the correct translations will entail the other. Label the stronger (i.e. entailing) sentence *Strong* and the weaker one *Weak*. [2 marks]

See above

(iii) Give a formal proof of *Weak* in $\mathcal{F}^+$, using *Strong* as your premise.
[8 marks]

1.	$\exists y\,(\text{Course}(y) \wedge \forall x\,(\text{Student}(x) \rightarrow \text{Taking}(x,y)))$	
2.	$\boxed{c}$ $\text{Course}(c) \wedge \forall x\,(\text{Student}(x) \rightarrow \text{Taking}(x,c))$	
3.	$\text{Course}(c)$	✓ ∧ Elim *:2*
4.	$\forall x\,(\text{Student}(x) \rightarrow \text{Taking}(x,c))$	✓ ∧ Elim *:2*
5.	$\boxed{a}$ $\text{Student}(a)$	
6.	$\text{Student}(a) \rightarrow \text{Taking}(a,c)$	✓ ∀ Elim *:4*
7.	$\text{Taking}(a,c)$	✓ → Elim *:5,6*
8.	$\text{Course}(c) \wedge \text{Taking}(a,c)$	✓ ∧ Intro *:3,7*
9.	$\exists y\,(\text{Course}(y) \wedge \text{Taking}(a,y))$	✓ ∃ Intro *:8*
10.	$\forall x\,(\text{Student}(x) \rightarrow \exists y\,(\text{Course}(y) \wedge \text{Taking}(x,y)))$	✓ ∀ Intro *:5-9*
11.	$\forall x\,(\text{Student}(x) \rightarrow \exists y\,(\text{Course}(y) \wedge \text{Taking}(x,y)))$	✓ ∃ Elim *:2-10,1*