THE UNIVERSITY OF BRITISH COLUMBIA

Philosophy 220A

Symbolic Logic I

ANSWERS TO FAKE FINAL EXAMINATION #2

TIME: 2½ HOURS

SPECIAL INSTRUCTIONS:

Answer all questions. *Write your answers in the separate answer booklet*. If you get stuck on a question, go on to the next, and return to it later. Indeed, it is wise to read the whole paper before you start, and begin with the easiest questions. Including this cover page, and the sheet of rules of inference, this examination booklet should consist of six pages. Check that these are all present before the examination begins.

INSTRUCTOR: Richard Johns

1.

(i) No cube is large. [2]

$\neg\exists x(Cube(x) \wedge Large(x))$

(ii) $\forall y(Tet(rm(y)) \rightarrow Dodec(y))$ [2]

Only dodecs have a tet as their rightmost object.

(iii) $\forall x\forall y\forall z((Dodec(x) \wedge Dodec(y) \wedge Dodec(z)) \rightarrow (x=y \vee y=z \vee x=z))$ [2]

There are at most two dodecs.

(iv) $\forall z(BackOf(z, a) \leftrightarrow LeftOf(z, a))$ [2]

The same objects are back of *a* as are left of *a*.

(v) If a dodec is smaller than a cube, then they're in the same row. [3]

$\forall x\forall y((Dodec(x) \wedge Cube(y) \wedge Smaller(x, y)) \rightarrow SameRow(x, y))$

(vi) The only cube in the world is small, unless it's medium. [3]

$\exists x(Cube(x) \wedge \forall y(Cube(y) \rightarrow x=y) \wedge (\neg Medium(x) \rightarrow Small(x)))$

(vii) The largest dodec is the leftmost object in its row [3]

$\exists x[Dodec(x) \wedge \forall y((Dodec(y) \wedge x\neq y) \rightarrow Larger(x, y)) \wedge x = lm(x)]$

(viii) Only tets and dodecs lie to the right of *a*. [3]

$\forall x(RightOf(x, a) \rightarrow (Tet(x) \vee Dodec(x)))$

(ix) $\exists x(Large(x) \wedge Tet(x) \wedge SameRow(x, a) \wedge \forall y((Large(y) \wedge Tet(y) \wedge SameRow(y, a)) \rightarrow x=y) \wedge Backof(x, b))$ [3]

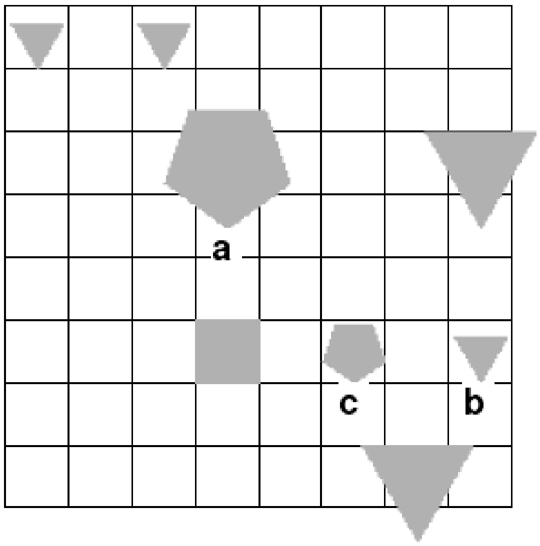
The large tet that is in the same row as *a* is back of *b*.

(x) Other than *b* and *c*, everything to the right of *a* is larger than something in back of *a*. [3]

$\forall x[(RightOf(x, a) \wedge x\neq b \wedge x\neq c) \rightarrow \exists y(BackOf(y, a) \wedge Larger(x, y))]$

2. Every sentence from Qu. 1, except one, is true in the world shown below. Which one sentence is false? [2 marks]

Sentence (ii) is false. The cube, for example, has a tet as its rightmost object.



3. Show that the following argument is valid by providing a proof in $\mathcal{F}^{+}$. [4 marks]

```
| 1. ¬(P → Q)
|----
| | 2. ¬P
| |----
| | | 3. P
| | |----
| | | 4. ⊥                      ⊥ Intro: 2,3
| | | 5. Q                      ⊥ Elim: 4
| |
| | 6. P → Q                    → Intro: 3-5
| | 7. ⊥                        ⊥ Intro: 6,1
|
| 8. P                          ¬ Intro: 2-7
| | 9. Q
| |----
| | | 10. P
| | |----
| | | 11. Q                     Reit: 9
| |
| | 12. P → Q                   → Intro: 10-11
| | 13. ⊥                       ⊥ Intro: 12,1
|
| 14. ¬Q                        ¬ Intro: 9-13
| 15. P ∧ ¬Q                    ∧ Intro: 8,14
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4. (i) For each of the two arguments (A) and (B) written below, decide whether the conclusion is a *logical consequence* of the premise(s). (Assume that the predicates have their usual meanings.) State your verdicts without justification. [4 marks]

A is logical con. B is logical con as well.

(A)

$\exists x \neg \mathrm{Cube}(x) \to \exists x\ \mathrm{Tet}(x)$
$\forall x(\mathrm{Tet}(x) \to x = a)$

$\neg \exists x \mathrm{Cube}(x) \to \mathrm{Tet}(a)$

(B)

$\forall x(\mathrm{Cube}(x) \to \mathrm{SameCol}(x, a))$
$\mathrm{Cube}(c)$

$\mathrm{SameCol}(a, c)$

(ii) Re-write both arguments (A) and (B), replacing all the non-logical predicates with nonsense predicates. I.e. write them as they appear through "first-order goggles". [4 marks]

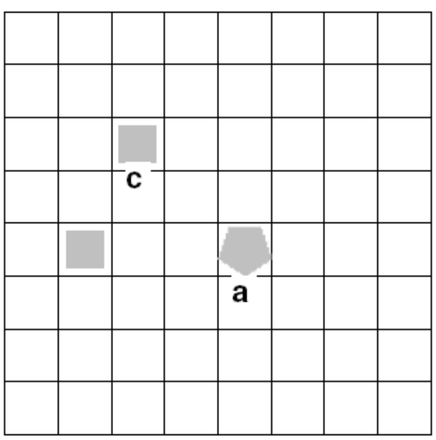
∃x¬Cubb(x) → ∃x Teot(x) ∀x(Teot(x) → x = a) ------------------- ¬∃x Cubb(x) → Teot(a)	∀x(Cubb(x) → SamCool(x, a)) Cubb(c) ------------------------- SamCool(a, c)

(iii) For each argument (A) and (B), say whether the conclusion is a first-order consequence of the premise(s). If an argument is FO con, then show this using a formal proof in $\mathcal{F}^+$. If it is not a first-order consequence, then show this by giving a suitable interpretation of the predicates, and constructing a suitable world. [8 marks]

A: FO con.
B: Not FO con.

```
1. ∃x ¬Cube(x) → ∃x Tet(x)
2. ∀x (Tet(x) → x = a)
   3. ¬∃x Cube(x)
      4. ¬∃x ¬Cube(x)
         5. ¬Cube(a)
         6. ∃x ¬Cube(x)                ∃ Intro: 5
         7. ⊥                          ⊥ Intro: 6,4
      8. Cube(a)                       ¬ Intro: 5-7
      9. ∃x Cube(x)                    ∃ Intro: 8
      10. ⊥                            ⊥ Intro: 9,3
   11. ∃x ¬Cube(x)                     ¬ Intro: 4-10
   12. ∃x Tet(x)                       → Elim: 1,11
      13. [b] Tet(b)
      14. Tet(b) → b = a               ∀ Elim: 2
      15. b = a                        → Elim: 14,13
      16. Tet(a)                       = Elim: 13,15
   17. Tet(a)                          ∃ Elim: 13-16,12
18. ¬∃x Cube(x) → Tet(a)               → Intro: 3-17
```

Let a Cubb be a cube, and SamCool(x, y) mean LeftOf(x, y).



(iv) If either argument above is FO con, then check whether it is also TT con by writing it in its truth-functional ("Boolean goggles") form, and constructing at least part of a truth table. State your verdict clearly, briefly justifying your answer using the truth table.
[5 marks]

A is FO con, so here's the Boolean goggles form:

$P \to Q$
R

$\neg S \to W$

Obviously it's not TT con. E.g. consider the row:

P	Q	R	S	W
T	T	T	F	F

5. In each of the following arguments, derive the conclusion from the premisses in $\mathcal{F}^+$. [5, 6, 8 marks]

a.

```
1. ∃x (Cube(x) → Large(b))
  2. ∀x Cube(x)
    3. [a] Cube(a) → Large(b)
    4. Cube(a)                      ✓ ∀ Elim: 2
    5. Large(b)                     ✓ → Elim: 3,4
  6. Large(b)                       ✓ ∃ Elim: 3-5,1
7. ∀x Cube(x) → Large(b)            ✓ → Intro: 2-6
```

b.

```
1. ∀y (Dog(y) → ∃x (Feeds(x, y) ∧ Likes(y, x)))
  2. [a] Dog(a)
  3. Dog(a) → ∃x (Feeds(x, a) ∧ Likes(a, x))    ✓ ∀ Elim: 1
  4. ∃x (Feeds(x, a) ∧ Likes(a, x))             ✓ → Elim: 2,3
    5. [b] Feeds(b, a) ∧ Likes(a, b)
    6. Feeds(b, a)                              ✓ ∧ Elim: 5
    7. ∃w Feeds(w, a)                           ✓ ∃ Intro: 6
  8. ∃w Feeds(w, a)                             ✓ ∃ Elim: 5-7,4
9. ∀z (Dog(z) → ∃w Feeds(w, z))                 ✓ ∀ Intro: 2-8
```

c. [*Hint: In this proof you can introduce any one sentence of the form* P ∨ ¬P, *citing "already shown"*.] **(I had to put 'Taut Con' in Fitch.)**

```
1. ∀x ((Dog(x) ∧ x ≠ a ∧ x ≠ b) → Larger(a, x))
2. Larger(b, a)
3. ∀x ∀y ∀z ((Larger(x, y) ∧ Larger(y, z)) → Larger(x, z))
   4. [c] Dog(c) ∧ c ≠ b
   5. (Dog(c) ∧ c ≠ a ∧ c ≠ b) → Larger(a, c)         ✓ ∀ Elim: 1
   6. c = a ∨ c ≠ a                                    ✓ Taut Con:
      7. c = a
      8. Larger(b, c)                                  ✓ = Elim: 7,2
      9. c ≠ a
      10. Dog(c) ∧ c ≠ a ∧ c ≠ b                       ✓ ∧ Intro: 4,9
      11. Larger(a, c)                                 ✓ → Elim: 10,5
      12. (Larger(b, a) ∧ Larger(a, c)) → Larger(b, c) ✓ ∀ Elim: 3
      13. Larger(b, a) ∧ Larger(a, c)                  ✓ ∧ Intro: 11,2
      14. Larger(b, c)                                 ✓ → Elim: 13,12
   15. Larger(b, c)                                    ✓ ∨ Elim: 6,7-8,9-14
16. ∀y ((Dog(y) ∧ y ≠ b) → Larger(b, y))               ✓ ∀ Intro: 4-15
```

6.

```
1. SameShape(a, b)
2. Cube(b)
3. ∀x ∀y ((SameShape(x, y) ∧ Tet(x)) → Tet(y))
4. ¬∃x (Cube(x) ∧ Tet(x))
   5. Tet(a)
   6. (SameShape(a, b) ∧ Tet(a)) → Tet(b)     ∀ Elim: 3
   7. SameShape(a, b) ∧ Tet(a)                ∧ Intro: 5,1
   8. Tet(b)                                  → Elim: 7,6
   9. Cube(b) ∧ Tet(b)                        ∧ Intro: 2,8
   10. ∃x (Cube(x) ∧ Tet(x))                  ∃ Intro: 9
   11. ⊥                                      ⊥ Intro: 10,4
12. ¬Tet(a)                                   ¬ Intro: 5-11
```

7. Abraham Lincoln said, "You can fool all of the people some of the time, and some of the people all the time, but you cannot fool all the people all the time." Let us focus on the first claim, that *you can fool all of the people some of the time*.

This sentence is ambiguous, and can be translated into FOL in two (non-equivalent) ways.

(i) Write down *both* translations, using the predicates CanFool(*x*, *t*), Person(*x*) and Time(*t*). [N.B. *Do not* use the shortcut I used in class, which was to assume that *x* ranges over persons, and *t* over times. You need to make this explicit in your sentence.]
[3 + 3 marks]

A: ∃t(Time(t) ∧ ∀x(Person(x) → CanFool(x, t)))

B: ∀x(Person(x) → ∃t(Time(t) ∧ CanFool(x, t)))

(ii) One translation should entail the other. Give a formal proof in $\mathcal{F}^+$ to show this. [6 marks]

1. ∃t(Time(t) ∧ ∀x (Person(x) → CanFool(x, t)))	
2. [a] Person(a)	
3. [b] Time(b) ∧ ∀x (Person(x) → CanFool(x, b))	
4. ∀x (Person(x) → CanFool(x, b))	∧ Elim: 3
5. Person(a) → CanFool(a, b)	∀ Elim: 4
6. CanFool(a, b)	→ Elim: 5,2
7. Time(b)	∧ Elim: 3
8. Time(b) ∧ CanFool(a, b)	∧ Intro: 7,6
9. ∃t(Time(t) ∧ CanFool(a, t))	∃ Intro: 8
10. ∃t(Time(t) ∧ CanFool(a, t))	∃ Elim: 3-9,1
11. ∀x (Person(x) → ∃t(Time(t) ∧ CanFool(x, t)))	∀ Intro: 2-10

(iii) Describe a world in which the premise of your argument is false, but the conclusion is true.
[2 marks]

The domain contains two people, Fred and Jim, and two times, Monday and Tuesday. Fred can be fooled on Monday only, and Jim can be fooled on Tuesday only.

(iv) Describe a world in which both the premise and conclusion of your argument are true.
[2 marks]

Same domain as part (iii). But now both Fred and Jim can be fooled on Monday.

(v) Fred disagrees with Lincoln, saying “You *cannot* fool all of the people some of the time”. This is an unfortunate sentence, since it is highly ambiguous, having *four* possible meanings! (At least four possible meanings, perhaps five or even six!) Express the four most plausible interpretations as separate FOL sentences. [4 marks]

????

$\neg\forall x(Person(x) \rightarrow \exists t(Time(t) \wedge CanFool(x, t)))$

$\neg\exists t(Time(t) \wedge \forall x(Person(x) \rightarrow CanFool(x, t)))$

$\forall x(Person(x) \rightarrow \neg\exists t(Time(t) \wedge CanFool(x, t)))$

$\exists t(Time(t) \wedge \neg\forall x(Person(x) \rightarrow CanFool(x, t)))$

$\forall x(Person(x) \rightarrow \exists t(Time(t) \wedge \neg CanFool(x, t)))$?

$\exists t(Time(t) \wedge \forall x(Person(x) \rightarrow \neg CanFool(x, t)))$?