Philosophy 220A, Section 001
Symbolic Logic I

ANSWERS TO THE FAKE MIDTERM

1. (i) Assuming *b* is large and *c* isn't, *b* is larger than *c*.

(Large(b) ∧ ¬Large(c)) → Larger(b, c)

(ii) Unless *a* is small, it is the same size as *c* just in case *c* isn't a tetrahedron.

¬Small(a) → (SameSize(a,c) ↔ ¬Tet(c))

(iii) (Cube(*a*) ↔ Small(*a*)) ∧ (Tet(*a*) → Medium(*a*))

a is a cube just in case it is small, and a tetrahedron only if it's medium.

2. [Total of 15 marks]

| Larger(a, b) ∨ Cube(c)
| a = b
|———
| ¬Tet(c)

(i) Is the argument above logically valid? _____Yes______ (Yes/ No)

(ii) Is the argument above TT valid? (I.e. is the conclusion a *tautological* consequence of the premisses?)

______No______ (Yes/ No)

(iii) If the answer to either question above is *No*, then demonstrate that this answer is correct by providing a world, or assignment of truth values to atomic sentences, as appropriate. (Write your answer in the space below.)

Larger(a, b)	Cube(c)	a=b	Tet(c)
T	F	T	T

(There are other assignments of truth values for which the premisses are all true, and the conclusion is false.)

3. Use a truth table to determine whether or not the following sentences are tautologically (TT) equivalent.

$(P \wedge \neg Q) \rightarrow R$ $\qquad \neg R \rightarrow (\neg Q \rightarrow \neg P)$

[12 marks for the table]

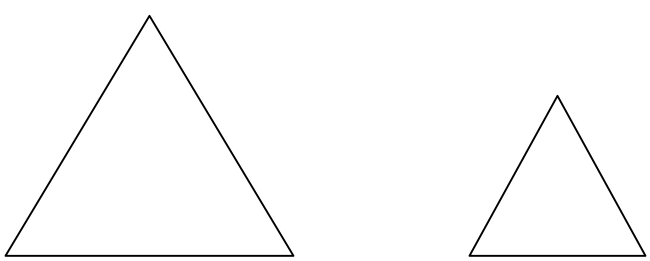
P	Q	R	$(P \wedge \neg Q) \rightarrow R$	$\neg R \rightarrow (\neg Q \rightarrow \neg P)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	**F**	**F**
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	T	T

Answer: ______They are TT equivalent__________ [2 marks]

4.

Medium(a) ∨ (¬Tet(a) → Small(a))
SameShape(a, b)
Smaller(b, a) ∨ (Cube(a) ∧ Dodec(b))
———
Small(b)

N.B. This is the only solution, if we restrict ourselves to Tarski's World worlds. Without that restriction (which is not stated in the question) there are other correct solutions.



a (large Tet) **b (medium Tet)**

5. For each of the following arguments, prove that the argument is valid by providing a formal proof (in $\mathcal{F}$) of the conclusion from the premises. [8, 9, 10, 11 marks]

(i)

1. $(C \lor G) \rightarrow (A \land B)$	
	2. C	
	3. $C \lor G$	$\lor$ Intro: 2
	4. $A \land B$	$\rightarrow$ Elim: 1,3
	5. A	$\land$ Elim: 4
6. $C \rightarrow A$	$\rightarrow$ Intro: 2-5

(ii)

1. $(D \land E) \rightarrow \neg F$	
2. $F \lor (G \land W)$	
3. $D \rightarrow E$	
	4. D	
	5. E	$\rightarrow$ Elim: 3,4
	6. $D \land E$	$\land$ Intro: 5,4
	7. $\neg F$	$\rightarrow$ Elim: 6,1
		8. F	
		9. $\bot$	$\bot$ Intro: 8,7
		10. G	$\bot$ Elim: 9
		11. $G \land W$	
		12. G	$\land$ Elim: 11
	13. G	$\lor$ Elim: 8-10,11-12,2
14. $D \rightarrow G$	$\rightarrow$ Intro: 4-13

(iii) Note how in this proof we start by assuming D, not ¬A, as this shortens the proof a little. Premiss 1 tells us that getting ¬D is as good as getting A. Moreover, if you look at premiss 2, you see that the assumption of D is very useful. Knowing ¬A is less useful.

1. $A \leftrightarrow \neg D$	
2. $(D \vee H) \rightarrow B$	
3. $\neg(B \vee G)$	
4. D	
5. $D \vee H$	**∨ Intro:** 4
6. B	**→ Elim:** 2,5
7. $B \vee G$	**∨ Intro:** 6
8. $\bot$	**⊥ Intro:** 7,3
9. $\neg D$	**¬ Intro:** 4-8
10. A	**↔ Elim:** 1,9