THE UNIVERSITY OF BRITISH COLUMBIA

Philosophy 220A

Symbolic Logic I

ANSWERS TO THE THIRD FAKE MIDTERM EXAMINATION

NAME: ____

STUDENT NUMBER: ______________________________________

SPECIAL INSTRUCTIONS:

Answer all questions. If you get stuck on a question, go on to the next, and return to it later. Indeed, it is wise to read the whole paper before you start, and begin with the easiest questions. Including this cover page, and the sheet of rules, this examination booklet should consist of eight pages. Check that these are all present before the examination begins.

Your answers to all questions should be written in this booklet, in the spaces provided.

For rough work, you may use the plain backs of the sheets in this booklet. If necessary, I can also supply a separate booklet for rough work.

SECTION 003
INSTRUCTOR: Richard Johns

1. Translate the following sentences from English to FOL, or FOL to English, using the predicates provided. [8, 8, 8 marks]

Cube(x)	Large(x)	Larger(x, y)	Adjoins(x, y)	Tet(x)
Medium(x)	Small(x)	SameRow(x, y)	Smaller(x, y)	

(i) *a* and *c* aren't both cubes, unless they're both larger than both *d* and *e*.

¬(Larger(a, d) ∧ Larger(a, e) ∧ Larger(c, d) ∧ Larger(c, e)) → ¬(Cube(a) ∧ Cube(c))

(ii) Provided *c* isn't smaller than *b*, *b* is larger than *a* only if *c* is larger than *a*.

¬Smaller(c, b) → (Larger(b, a) → Larger(c, a))

(iii) (¬Small(*a*) ∨ ¬Cube(*a*)) → ((Large(*a*) ∨ Medium(*a*)) ∧ Tet(*a*))

If a is either not small or not a cube, it's either a large or medium tet.

2.

$$\begin{array}{|l} \neg(\text{Cube}(c) \to \text{Dodec}(c)) \\ \neg\text{Dodec}(c) \to \text{Small}(c) \\ \hline \text{Adjoins}(c, d) \to \text{Small}(c) \end{array} \qquad \begin{array}{|l} \text{FrontOf}(d, c) \lor \text{Smaller}(d, c) \\ c = d \lor \text{Cube}(c) \\ \neg(\text{Cube}(c) \land \text{Dodec}(c)) \\ \hline \neg\text{Dodec}(c) \end{array}$$

TT con?	**Yes**	**No**
Logical con?	**Yes**	**Yes**

(i) Answer the four questions above, filling the spaces provided with 'yes' or 'no'. [8 marks]

(ii) Show that one of the arguments is not TT valid by means of *one row* of a truth table, i.e. by providing *one* suitable assignment of truth values to the atomic sentences within the argument. (If space is tight then just do the reference columns.) [8]

FrontOf(d, c)	Smaller(d, c)	c = d	Cube(c)	Dodec(c)
T	T	T	F	T

3. Use a truth table to determine whether or not the following argument is a tautological (TT) consequence. Place an asterisk (*) next to any row that is sufficient to determine the answer. (There may be no such row. If there is more than one, then mark them all.)
[8 marks for table]

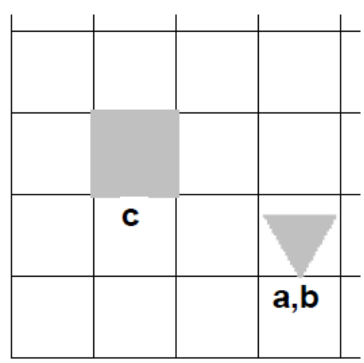
$$\begin{array}{|l} P \leftrightarrow (Q \land R) \\ \hline (P \leftrightarrow Q) \land (P \leftrightarrow R) \end{array}$$

	P	Q	R	$P \leftrightarrow (Q \land R)$	$(P \leftrightarrow Q) \land (P \leftrightarrow R)$
	T	T	T	**T**	**T**
	T	T	F	F	F
	T	F	T	F	F
	T	F	F	F	F
	F	T	T	F	F
*	F	T	F	**T**	F
*	F	F	T	**T**	F
	F	F	F	**T**	**T**

Answer: Not TT con [6 marks]

4. Show that the following argument is not logically valid, by constructing an appropriate world. Assume that the rules of "Tarski's World" apply. [12 marks]

$$\begin{array}{|l} \text{RightOf}(a, c) \rightarrow \text{Smaller}(a, c) \\ \text{Medium}(c) \leftrightarrow (\text{Cube}(c) \land \text{Tet}(b)) \\ \text{Small}(c) \rightarrow \text{LeftOf}(a, c) \\ \neg(\text{Large}(c) \lor \text{Large}(a)) \\ \hline \text{RightOf}(a, c) \rightarrow a \neq b \end{array}$$



5. For each of the following arguments, prove that the argument is valid by providing a formal proof (in $\mathcal{F}$) of the conclusion from the premises.

(i) Write your proof under the premises provided below. [8 marks]

| Small(c) → ¬Small(a)
| Small(a)
|—
| a ≠ c

| Small(c) → ¬Small(a)
| Small(a)
|—

1. Small(c) → ¬Small(a)	
2. Small(a)	
3. a = c	
4. Small(c)	= Elim :*2,3*
5. ¬Small(a)	→ Elim :*1,4*
6. ⊥	⊥ Intro :*5,4*
7. ¬a = c	¬ Intro :*3-6*

(ii) Write your proof under the premises provided below. [8 marks]

$$\begin{array}{|l} A \rightarrow (B \wedge C) \\ \hline \neg B \rightarrow \neg A \end{array} \qquad\qquad \begin{array}{|l} A \rightarrow (B \wedge C) \\ \hline \\ \end{array}$$

1.	$A \rightarrow (B \wedge C)$	
2.	$\neg B$	
3.	A	
4.	$B \wedge C$	✓ $\rightarrow$ Elim :*1,3*
5.	B	✓ $\wedge$ Elim :*4*
6.	$\perp$	✓ $\perp$ Intro :*5,2*
7.	$\neg A$	✓ $\neg$ Intro :*3-6*
8.	$\neg B \rightarrow \neg A$	✓ $\rightarrow$ Intro :*2-7*

(iii) Write your proof under the premises provided below. [8 marks]

$\neg(D \vee G)$
$B \rightarrow A$
$(D \vee G) \vee (C \rightarrow A)$

$(B \vee C) \rightarrow A$

1. $\neg(D \vee G)$	
2. $B \rightarrow A$	
3. $D \vee G \vee (C \rightarrow A)$	
4. $B \vee C$	
5. B	
6. A	✓ $\rightarrow$ Elim *:2,5*
7. C	
8. $D \vee G$	
9. $\perp$	✓ $\perp$ Intro *:8,1*
10. A	✓ $\perp$ Elim *:9*
11. $C \rightarrow A$	
12. A	✓ $\rightarrow$ Elim *:11,7*
13. A	✓ $\vee$ Elim *:3,8-10,11-12*
14. A	✓ $\vee$ Elim *:4,5-6,7-13*
15. $(B \vee C) \rightarrow A$	✓ $\rightarrow$ Intro *:4-14*

(iv) Write your proof under the premises provided below. In this proof you may introduce one sentence of the form **P** $\vee$ $\neg$**P** (where **P** is any FOL sentence) without proof. Cite it as 'already shown'. [10 marks]

$A \leftrightarrow B$

$(A \wedge B) \vee (\neg A \wedge \neg B)$

1. $A \leftrightarrow B$	
2. $A \vee \neg A$	Theorem 6.33
3. A	
4. B	$\leftrightarrow$ Elim :*1,3*
5. $A \wedge B$	$\wedge$ Intro :*3,4*
6. $(A \wedge B) \vee (\neg A \wedge \neg B)$	$\vee$ Intro :*5*
7. $\neg A$	
8. B	
9. A	$\leftrightarrow$ Elim :*8,1*
10. $\perp$	$\perp$ Intro :*9,7*
11. $\neg B$	$\neg$ Intro :*8-10*
12. $\neg A \wedge \neg B$	$\wedge$ Intro :*7,11*
13. $(A \wedge B) \vee (\neg A \wedge \neg B)$	$\vee$ Intro :*12*
14. $(A \wedge B) \vee (\neg A \wedge \neg B)$	$\vee$ Elim :*2,3-6,7-13*