THE UNIVERSITY OF BRITISH COLUMBIA

Philosophy 220A

Symbolic Logic I

SOLUTIONS TO THE OTHER FAKE MIDTERM EXAMINATION

TIME: 75 MINUTES

NAME: _____

STUDENT NUMBER: _____

SPECIAL INSTRUCTIONS:

Answer all questions. If you get stuck on a question, go on to the next, and return to it later. Indeed, it is wise to read the whole paper before you start, and begin with the easiest questions. Including this cover page, and the sheet of rules, this examination booklet should consist of eight pages. Check that these are all present before the examination begins.

Your answers to all questions should be written in this booklet, in the spaces provided.

For rough work, you may use the plain backs of the sheets in this booklet. If necessary, I can also supply a separate booklet for rough work.

INSTRUCTOR: Richard Johns

1. Translate the following sentences from English to FOL, or FOL to English, using the dictionary provided. [8, 8, 8 marks]

Cube(x) Large(x) Larger(x, y) Adjoins(x,y) Tet(x)
Medium(x) Small(x) SameRow(x, y)

(i) a is small, but it's a cube only if it's in the same row as either b or c.

$\text{Small}(a) \wedge (\text{Cube}(a) \rightarrow (\text{SameRow}(a, b) \vee \text{SameRow}(a, c)))$

(ii) Unless it's medium, b is small if and only if it is not large.

$\neg\text{Medium}(b) \rightarrow (\text{Small}(b) \leftrightarrow \neg\text{Large}(b))$

(iii) $\neg(\text{Small}(a) \wedge \text{Cube}(a)) \rightarrow ((\text{Large}(a) \vee \text{Medium}(a)) \wedge \text{Tet}(a))$

Unless *a* is a small cube, it's either a large or a medium tet.

2.

$$\begin{array}{l} \text{Cube(a)} \\ \text{Large(b)} \rightarrow \text{Cube(b)} \\ \hline \text{Large(b)} \rightarrow \text{SameShape(a, b)} \end{array}$$

$$\begin{array}{l} \text{Cube(a)} \rightarrow (\text{Dodec(b)} \rightarrow \text{a} \neq \text{b}) \\ \text{Dodec(b)} \wedge \text{a} = \text{b} \\ \hline \neg\text{Cube(a)} \end{array}$$

TT valid?	_____NO____	_____YES____
Logically valid?	___YES_____	_____YES__

(i) Answer the four questions above, filling the spaces provided with 'yes' or 'no'. [8 marks]

(ii) Show that one of the arguments is not TT valid by means of *one row* of a truth table, i.e. by providing *one* suitable assignment of truth values to the atomic sentences within the argument. [8]

Cube(a)	Large(b)	Cube(b)	SameShape(a,b)
T	T	T	F

N.B. I have only given the reference columns here. That's all you need. In this row, all three premises are true and the conclusion is false.

3. Use a truth table to determine whether or not the conclusion of the following argument is a tautological (TT) consequence of the premises. Place an asterisk (*) next to any row (or rows) that is sufficient to determine the answer. (There may be no such row.) [8 marks for table]

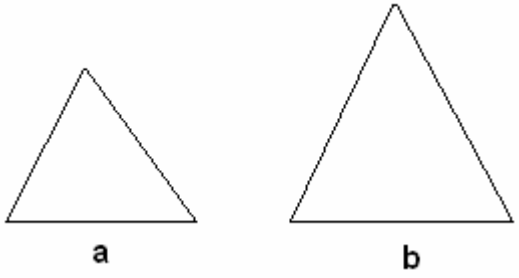
$$\begin{array}{l} A \rightarrow (H \wedge J) \\ J \leftrightarrow H \\ \neg J \\ \hline \neg A \end{array}$$

A	H	J	A→(H∧J)	J↔H	¬J	¬A
T	T	T	T	T	F	F
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	F	F	T
F	F	F	T	T	T	T

Answer: _____TT Con_____ [6 marks]

4. Show that the following argument is not logically valid, by constructing an appropriate world. [12 marks]

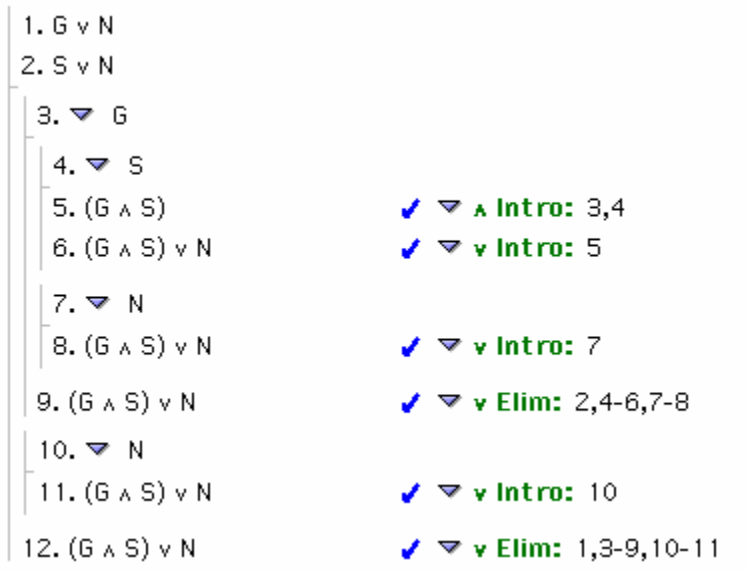
$$\begin{array}{l} \text{Tet}(a) \rightarrow \text{SameRow}(a, b) \\ \neg\text{SameSize}(a, b) \wedge \text{RightOf}(b, a) \\ \text{Smaller}(b, a) \vee \text{Tet}(a) \\ \hline \text{Larger}(a, b) \vee \neg\text{SameShape}(a, b) \end{array}$$



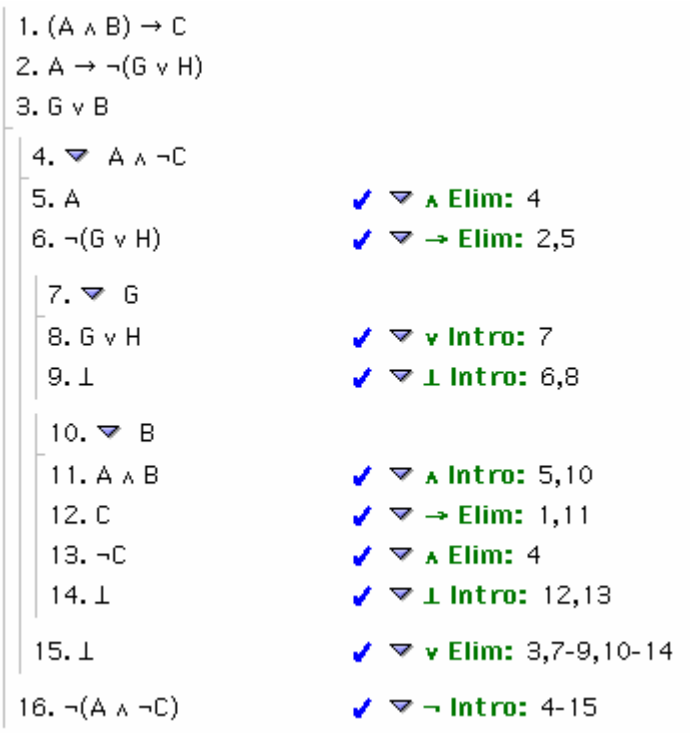
5. For each of the following arguments, prove that the argument is valid by providing a formal proof (in $\mathcal{F}$) of the conclusion from the premises.

(i) [6 marks]

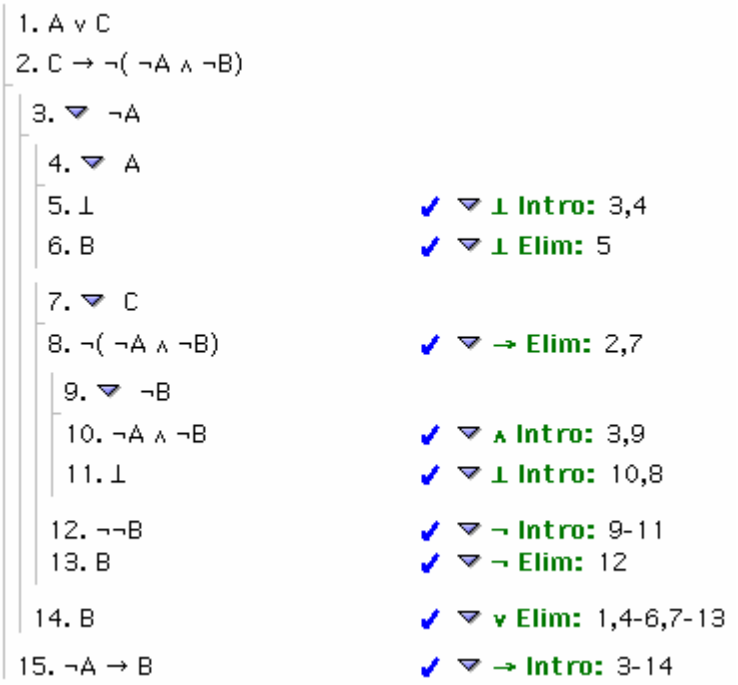
(ii) [8 marks]



(iii) [8 marks]


1. (A ∧ B) → C
2. A → ¬(G ∨ H)
3. G ∨ B
4. A ∧ ¬C
5. A ∧ Elim: 4
6. ¬(G ∨ H) → Elim: 2,5
7. G
8. G ∨ H ∨ Intro: 7
9. ⊥ ⊥ Intro: 6,8
10. B
11. A ∧ B ∧ Intro: 5,10
12. C → Elim: 1,11
13. ¬C ∧ Elim: 4
14. ⊥ ⊥ Intro: 12,13
15. ⊥ ∨ Elim: 3,7-9,10-14
16. ¬(A ∧ ¬C) ¬ Intro: 4-15

(iv) . [10 marks]


1. A ∨ C
2. C → ¬(¬A ∧ ¬B)
3. ¬A
4. A
5. ⊥ ⊥ Intro: 3,4
6. B ⊥ Elim: 5
7. C
8. ¬(¬A ∧ ¬B) → Elim: 2,7
9. ¬B
10. ¬A ∧ ¬B ∧ Intro: 3,9
11. ⊥ ⊥ Intro: 10,8
12. ¬¬B ¬ Intro: 9-11
13. B ¬ Elim: 12
14. B ∨ Elim: 1,4-6,7-13
15. ¬A → B → Intro: 3-14