ANSWERS TO FAKE FINAL EXAMINATION

1. (i) $P \wedge Q$

$R \rightarrow S$

(ii)

P	Q	R	S	$P \wedge Q$	$R \rightarrow S$
T	T	T	F	T	F

Not TT con. In this row, the premise is true but the conclusion is false.

(iii)

$\exists y\, Trat(y) \wedge \forall z(Trat(z) \rightarrow \forall w\, Lig(w))$

$Crub(a) \rightarrow Lig(a)$

(iv) It is FO con, so we do a proof.

1. $\exists y\, Trat(y) \wedge \forall z\, (Trat(z) \rightarrow \forall w\, Lig(w))$	
2. $Crub(a)$	
3. $\exists y\, Trat(y)$	$\wedge$ Elim: 1
4. $\forall z\, (Trat(z) \rightarrow \forall w\, Lig(w))$	$\wedge$ Elim: 1
5. [b] $Trat(b)$	
6. $Trat(b) \rightarrow \forall w\, Lig(w)$	$\forall$ Elim: 4
7. $\forall w\, Lig(w)$	$\rightarrow$ Elim: 5,6
8. $Lig(a)$	$\forall$ Elim: 7
9. $Lig(a)$	$\exists$ Elim: 5-8,3
10. $Crub(a) \rightarrow Lig(a)$	$\rightarrow$ Intro: 2-9

2.

(i) The mother of Conrad is smaller than him. [2]
Smaller(mother(conrad), conrad)

(ii) Only large cubes are left of b. [2]
$\forall$x(LeftOf(x,b) $\rightarrow$ (Large(x) $\wedge$ Cube(x)))

(iii) $\forall z$(LeftOf(z, a) $\leftrightarrow$ BackOf(z, b)) [2]
The same things are left of a as are back of b.
(A thing is left of a if and only if it's back of b.)

(iv) $\exists w$(Tet(w) $\wedge$ Smaller(w, c) $\wedge$ $\forall x$((Tet(x) $\wedge$ Smaller(x, c)) $\rightarrow$ x=w) $\wedge$ SameCol(w, d)) [2]
The tet that is smaller than c is in the same column as d.

(v) The smallest cube is in back of b. [2]
$\exists$x(Cube(x) $\wedge$ $\forall$y((Cube(y) $\wedge$ x$\neq$y) $\rightarrow$ Smaller(x, y)) $\wedge$ BackOf(x, b))

(vi) If a tetrahedron has any cube to the left of it, then it (the tetrahedron) is large. [3]
$\forall$x$\forall$y((Tet(x) $\wedge$ Cube(y) $\wedge$ LeftOf(y, x)) $\rightarrow$ Large(x))
or: $\forall$x((Tet(x) $\wedge$ $\exists$y(Cube(y) $\wedge$ LeftOf(y, x))) $\rightarrow$ Large(x))

[Note that these are equivalent.]

(vii) $\neg\exists x\exists y\exists z$($y\neq z$ $\wedge$ Between(x, y, z) $\wedge$ SameShape(y, z)) [3]
Nothing is between two things that are the same shape.

(viii) Cube(a) $\wedge$ $\forall$y(Cube(y) $\rightarrow$ y = a) $\wedge$ Large(a) [3]
a is the only cube and it's large.

(ix) $\exists x$(BackOf(x, b) $\wedge$ $\exists y\exists z$($y\neq z$ $\wedge$ BackOf(y, x) $\wedge$ BackOf(z, x)) [3]
There are two things back of something that is back of b.

(x) $\forall x$(Loves(x, conrad) $\rightarrow$ (x = conrad $\vee$ x = mother(conrad))) [3]
No one, other than Conrad himself and his mother, loves Conrad.

3.

1.	$\neg(D \wedge \neg B)$		
2.	$(A \wedge E) \rightarrow B$		
3.	$A \rightarrow (D \vee E)$		
4.	A		
5.	$D \vee E$	→ Elim	4,3
6.	D		
7.	$\neg B$		
8.	$D \wedge \neg B$	∧ Intro	6,7
9.	$\bot$	⊥ Intro	1,8
10.	B	¬ Intro	7-9
11.	E		
12.	$A \wedge E$	∧ Intro	4,11
13.	B	→ Elim	2,12
14.	B	∨ Elim	5,6-10,11-13
15.	$A \rightarrow B$	→ Intro	4-14

4. (i) Is the conclusion of the following argument a logical consequence of the premisses? (Assume that the predicates have their usual meanings.) [2 marks]

Yes.

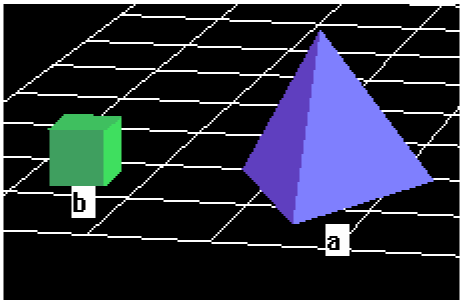
$\forall x(\text{Cube}(x) \rightarrow \text{Adjoins}(x, a))$
Cube(b)
____
Adjoins(a, b)

(ii)
∀x(Cabb(x) → Ajjo(x, a))
Cabb(b)

Ajjo(a, b)

(iii)

Not a FO consequence. Let Cabb(x) mean Cube(x), and Ajjo(x, y) means LeftOf(x, y). Then here's a counterexample world:



T	1. ∀x (Cube(x) → LeftOf(x, a))
T	2. Cube(b)
F	3. LeftOf(a, b)

5.

a.

1. ∀x (Cube(x) → Large(x))
2. ∃x (Adjoins(x, b) ∧ Large(x)) ∧ ∃x (Adjoins(x, b) ∧ ¬Large(x))

3. ∀x (Adjoins(x, b) → Cube(x))	
4. ∃x (Adjoins(x, b) ∧ ¬Large(x))	∧ Elim: 2
5. [a] Adjoins(a, b) ∧ ¬Large(a)	
6. Cube(a) → Large(a)	∀ Elim: 1
7. ¬Large(a)	∧ Elim: 5
8. ¬Cube(a)	MT: 6,7
9. Adjoins(a, b)	∧ Elim: 5
10. Adjoins(a, b) → Cube(a)	∀ Elim: 3
11. Cube(a)	→ Elim: 9,10
12. ⊥	⊥ Intro: 8,11
13. ⊥	∃ Elim: 5-12,4
14. ¬∀x (Adjoins(x, b) → Cube(x))	¬ Intro: 3-13

b.

1. ∃x(Boy(x) ∧ ∀y(Girl(y) → Likes(y, x)))	
2. ∀y∀z(Likes(y, z) → Likes(z, y))	
3. [a] Girl(a)	
4. [b] Boy(b) ∧ ∀y (Girl(y) → Likes(y, b))	
5. Boy(b)	∧ Elim: 4
6. ∀y (Girl(y) → Likes(y, b))	∧ Elim: 4
7. Girl(a) → Likes(a, b)	∀ Elim: 6
8. Likes(a, b)	→ Elim: 3,7
9. ∀z (Likes(a, z) → Likes(z, a))	∀ Elim: 2
10. Likes(a, b) → Likes(b, a)	∀ Elim: 9
11. Likes(b, a)	→ Elim: 8,10
12. ∃z Likes(z, a)	∃ Intro: 11
13. ∃z Likes(z, a)	∃ Elim: 4-12,1
14. ∀y (Girl(y) → ∃z Likes(z, y))	∀ Intro: 3-13

c.

1. ∃x (Cube(x) ∧ ∀y (Cube(y) → x = y) ∧ Large(x))	
2. ¬Large(a)	
3. Cube(a)	
4. [b] Cube(b) ∧ ∀y (Cube(y) → b = y) ∧ Large(b)	
5. Cube(b)	∧ Elim: 4
6. ∀y (Cube(y) → b = y)	∧ Elim: 4
7. Cube(a) → b = a	∀ Elim: 6
8. b = a	→ Elim: 7,3
9. Large(b)	∧ Elim: 4
10. Large(a)	= Elim: 8,9
11. ⊥	⊥ Intro: 2,10
12. ⊥	∃ Elim: 1,4-11
13. ¬Cube(a)	¬ Intro: 3-12

6.
Note that I've entered the axioms A2, A5, A6 and A7 as premises, in order to cite them in Fitch. You don't have to write them out, however.

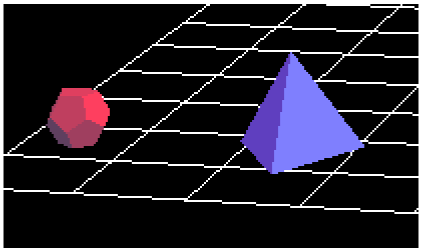
1. a = b	
2. ∀x (Tet(x) ∨ Dodec(x) ∨ Cube(x))	
3. ∀x ∀y ((Tet(x) ∧ Tet(y)) → SameShape(x, y))	
4. ∀x ∀y ((Dodec(x) ∧ Dodec(y)) → SameShape(x, y))	
5. ∀x∀y ((Cube(x) ∧ Cube(y)) → SameShape(x, y))	
6. Tet(a) ∨ Dodec(a) ∨ Cube(a)	∀ Elim: 2
7. Tet(a)	
8. Tet(b)	= Elim: 7,1
9. (Tet(a) ∧ Tet(b)) → SameShape(a, b)	∀ Elim: 3
10. Tet(a) ∧ Tet(b)	∧ Intro: 7,8
11. SameShape(a, b)	→ Elim: 9,10
12. Dodec(a)	
13. Dodec(b)	= Elim: 12,1
14. Dodec(a) ∧ Dodec(b)	∧ Intro: 12,13
15. (Dodec(a) ∧ Dodec(b)) → SameShape(a, b)	∀ Elim: 4
16. SameShape(a, b)	→ Elim: 15,14
17. Cube(a)	
18. Cube(b)	= Elim: 17,1
19. Cube(a) ∧ Cube(b)	∧ Intro: 17,18
20. (Cube(a) ∧ Cube(b)) → SameShape(a, b)	∀ Elim: 5
21. SameShape(a, b)	→ Elim: 19,20
22. SameShape(a, b)	∨ Elim: 6,7-11,12-16

7.

(i) ∀x∀y((Tet(x) ∧ Cube(y) ∧ Adjoins(x, y)) → Larger(x, y))

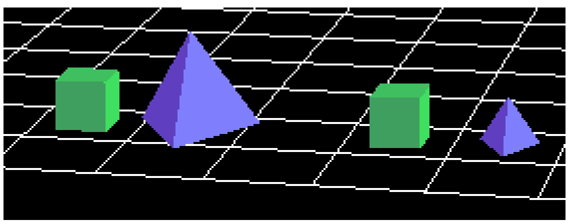
∃y(Cube(y) ∧ ∀x((Tet(x) ∧ Adjoins(x, y)) → Larger(x, y)))

(ii)



T 1. $\forall y\, \forall x\, ((\mathrm{Tet}(x) \land \mathrm{Cube}(y) \land \mathrm{Adjoins}(x, y)) \rightarrow \mathrm{Larger}(x, y))$

F 2. $\exists y\, (\mathrm{Cube}(y) \land \forall x\, ((\mathrm{Tet}(x) \land \mathrm{Adjoins}(x, y)) \rightarrow \mathrm{Larger}(x, y)))$



F 1. $\forall y\, \forall x\, ((\mathrm{Tet}(x) \land \mathrm{Cube}(y) \land \mathrm{Adjoins}(x, y)) \rightarrow \mathrm{Larger}(x, y))$

T 2. $\exists y\, (\mathrm{Cube}(y) \land \forall x\, ((\mathrm{Tet}(x) \land \mathrm{Adjoins}(x, y)) \rightarrow \mathrm{Larger}(x, y)))$

(iii) The sentence “All politicians are not corrupt” is also ambiguous. Write down two (non-equivalent) translations of it into FOL, using the predicates Politician(*x*) and Corrupt(*x*). [5 marks]

$\forall x(\mathrm{Politician}(x) \rightarrow \neg\mathrm{Corrupt}(x))$

$\neg\forall x(\mathrm{Politician}(x) \rightarrow \mathrm{Corrupt}(x))$