THE UNIVERSITY OF BRITISH COLUMBIA

Philosophy 220A

Symbolic Logic I

FAKE FINAL EXAMINATION

TIME: 2½ HOURS

SPECIAL INSTRUCTIONS:

Answer all questions. Write all your answers in the separate answer booklet. This question booklet must not contain any answers, and must also be handed in before you leave. If you get stuck on a question, go on to the next, and return to it later. Indeed, it is wise to read the whole paper before you start, and begin with the easiest questions. Including this cover page, and the sheet of rules of inference, this examination booklet should consist of five numbered sides, on three sheets of paper. Check that these are all present before the examination begins.

INSTRUCTOR: Richard Johns

1. (i) Write the "Boolean goggles" form of the following argument. [4 marks]

$$\begin{array}{|l} \exists y\,\mathrm{Tet}(y) \wedge \forall z(\mathrm{Tet}(z) \rightarrow \forall w\,\mathrm{Large}(w)) \\ \hline \mathrm{Cube(a)} \rightarrow \mathrm{Large(a)} \end{array}$$

(ii) Use a *truth table* to determine whether or not the conclusion of this argument is a tautological (TT) consequence of the premiss. State your verdict clearly. Note that you need only complete as much of the table as is needed to determine the answer. [4 marks]

(iii) Now write the argument as it appears with "first-order goggles" on, i.e. use the replacement method to write down the argument with "nonsense predicates". [4 marks]

(iv) Is the conclusion of the argument a first-order consequence of the premises? If so, then show this using a formal proof in $\mathcal{F}^+$. If it is not, then show this using a suitable interpretation of the nonsense predicates and a counterexample world. [8 marks]

2. Translate the following English sentences into FOL, and the FOL sentences into good, simple English. Take the set of all objects as your domain of discourse, and use *only* the predicates and function listed below. [Total 25 marks]

Cube(x)	Large(x)	Tet(x)	Smaller(x, y)	LeftOf(x, y)
SameCol (x, y)	BackOf(x, y)	Between(x, y, z)	Loves(x, y)	mother(x)

(i) The mother of Conrad is smaller than him. [2]

(ii) Only large cubes are left of b. [2]

(iii) $\forall z(\mathrm{LeftOf}(z, \mathrm{a}) \leftrightarrow \mathrm{BackOf}(z, \mathrm{b}))$ [2]

(iv) $\exists w(\mathrm{Tet}(w) \wedge \mathrm{Smaller}(w, \mathrm{c}) \wedge \forall x((\mathrm{Tet}(x) \wedge \mathrm{Smaller}(x, \mathrm{c})) \rightarrow x{=}w) \wedge \mathrm{SameCol}(w, \mathrm{d}))$ [2]

(v) The smallest cube is in back of b. [2]

(vi) If a tetrahedron has any cube to the left of it, then it (the tetrahedron) is large. [3]

(vii) $\neg\exists x\exists y\exists z(y \neq z \wedge \mathrm{Between}(x, y, z) \wedge \mathrm{SameShape}(y, z))$ [3]

(viii) $\mathrm{Cube(a)} \wedge \forall \mathrm{y}(\mathrm{Cube(y)} \rightarrow \mathrm{y = a}) \wedge \mathrm{Large(a)}$ [3]

(ix) $\exists x(\mathrm{BackOf}(x, \mathrm{b}) \wedge \exists y\exists z(y{\neq}z \wedge \mathrm{BackOf}(y, x) \wedge \mathrm{BackOf}(z, x))$ [3]

(x) $\forall x(\mathrm{Loves}(x, \mathrm{conrad}) \rightarrow (x = \mathrm{conrad} \vee x = \mathrm{mother(conrad)}))$ [3]

3. Show that the following argument is valid by providing a proof in $\mathcal{F}^+$. [4 marks]

$$\begin{array}{|l} \neg(D \wedge \neg B) \\ (A \wedge E) \to B \\ \underline{A \to (D \vee E)} \\ A \to B \end{array}$$

4. (i) Is the conclusion of the following argument a logical consequence of the premisses? (Assume that the predicates have their usual meanings.) [2 marks]

$$\begin{array}{|l} \forall x(\mathrm{Cube}(x) \to \mathrm{Adjoins}(x, a)) \\ \underline{\mathrm{Cube}(b)} \\ \mathrm{Adjoins}(a, b) \end{array}$$

(ii) Re-write the argument as it appears wearing "first-order goggles", i.e. replacing all the non-logical predicates with meaningless predicates. [2 marks]

(iii) If the conclusion is a first-order consequence of the premiss, then show this using a formal proof in $\mathcal{F}^+$. If it is not a first-order consequence, then show this by giving a suitable interpretation of the nonsense predicates, and constructing a suitable world. [5 marks]

5. In each of the following arguments, derive the conclusion from the premisses in $\mathcal{F}^+$. [5, 6, 8 marks]

a.

$$\begin{array}{|l} \forall x(\mathrm{Cube}(x) \to \mathrm{Large}(x)) \\ \underline{\exists x(\mathrm{Adjoins}(x, b) \wedge \mathrm{Large}(x)) \wedge \exists x(\mathrm{Adjoins}(x, b) \wedge \neg\mathrm{Large}(x))} \\ \neg\forall x(\mathrm{Adjoins}(x, b) \to \mathrm{Cube}(x)) \end{array}$$

b.

$$\begin{array}{|l} \exists x(\mathrm{Cube}(x) \wedge \forall y(\mathrm{Cube}(y) \to x=y) \wedge \mathrm{Large}(x)) \\ \underline{\neg\mathrm{Large}(a)} \\ \neg\mathrm{Cube}(a) \end{array}$$

c.

$$\begin{array}{|l} \exists x[\mathrm{Boy}(x) \wedge \forall y(\mathrm{Girl}(y) \to \mathrm{Likes}(y, x))] \\ \underline{\forall y \forall z(\mathrm{Likes}(y, z) \to \mathrm{Likes}(z, y))} \\ \forall y(\mathrm{Girl}(y) \to \exists z\, \mathrm{Likes}(z, y)) \end{array}$$

6. The following argument is logically valid, but the conclusion is not a first-order consequence of the premisses. The conclusion may be derived in $\mathcal{F}^+$, however, using additional premisses from among the ten shape axioms provided. First add the required axioms to the premisses of the argument, and then derive the conclusion in $\mathcal{F}^+$. [8 marks]

Shape Axioms.

1. $\neg\exists x\,(\text{Cube}(x) \wedge \text{Tet}(x))$
2. $\neg\exists x\,(\text{Tet}(x) \wedge \text{Dodec}(x))$
3. $\neg\exists x\,(\text{Dodec}(x) \wedge \text{Cube}(x))$
4. $\forall x\,(\text{Tet}(x) \vee \text{Dodec}(x) \vee \text{Cube}(x))$
5. $\forall x\,\forall y\,((\text{Cube}(x) \wedge \text{Cube}(y)) \rightarrow \text{SameShape}(x, y))$
6. $\forall x\,\forall y\,((\text{Dodec}(x) \wedge \text{Dodec}(y)) \rightarrow \text{SameShape}(x, y))$
7. $\forall x\,\forall y\,((\text{Tet}(x) \wedge \text{Tet}(y)) \rightarrow \text{SameShape}(x, y))$
8. $\forall x\,\forall y\,((\text{SameShape}(x, y) \wedge \text{Cube}(x)) \rightarrow \text{Cube}(y))$
9. $\forall x\,\forall y\,((\text{SameShape}(x, y) \wedge \text{Dodec}(x)) \rightarrow \text{Dodec}(y))$
10. $\forall x\,\forall y\,((\text{SameShape}(x, y) \wedge \text{Tet}(x)) \rightarrow \text{Tet}(y))$

$$\begin{array}{|l} a = b \\ \hline \text{SameShape}(a, b) \end{array}$$

7. Consider the sentence "Every tetrahedron that adjoins a cube is larger than that cube".

This sentence is ambiguous, and can be translated into FOL in two (non-equivalent) ways. (The issue is whether 'a cube' refers to one special cube in the world, or whether it can be a different cube for each tetrahedron.)

(i) Write down both translations. [3 + 3 marks]

(ii) Neither translation should entail the other. Thus, for each of the two FOL sentences, you should be able to construct a world in which that sentence is true and the other one is false. Draw two such worlds. [4 marks]

(iii) The sentence "All politicians are not corrupt" is also ambiguous. Write down two (non-equivalent) translations of it into FOL, using the predicates Politician(x) and Corrupt(x). [5 marks]

END OF EXAM

[Total: 100 marks]