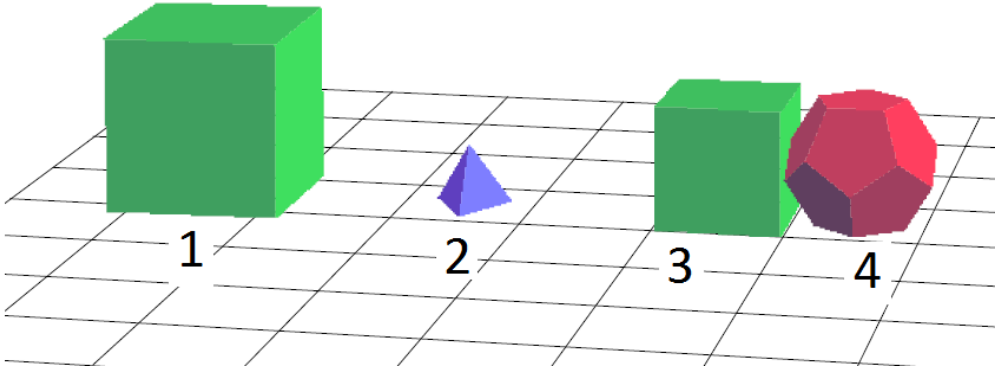
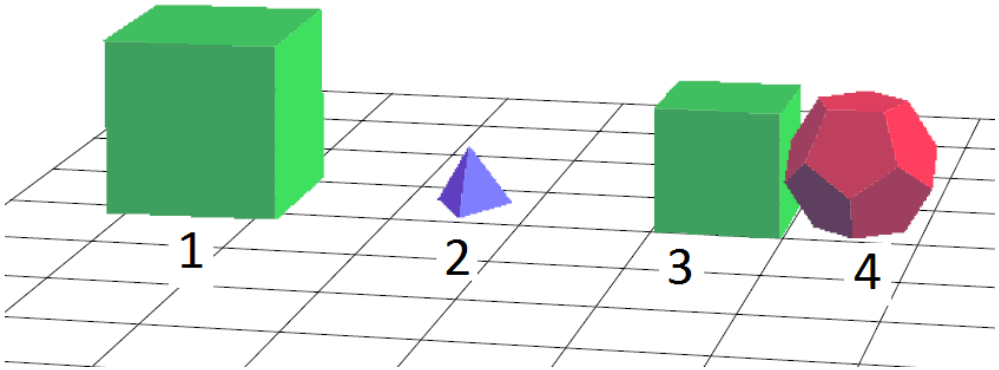
Satisfaction tables

Including multiple quantifiers

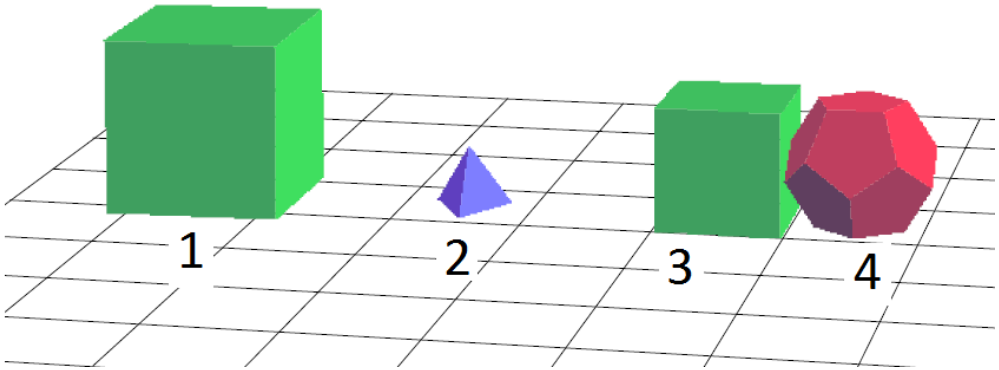
Simple cases …



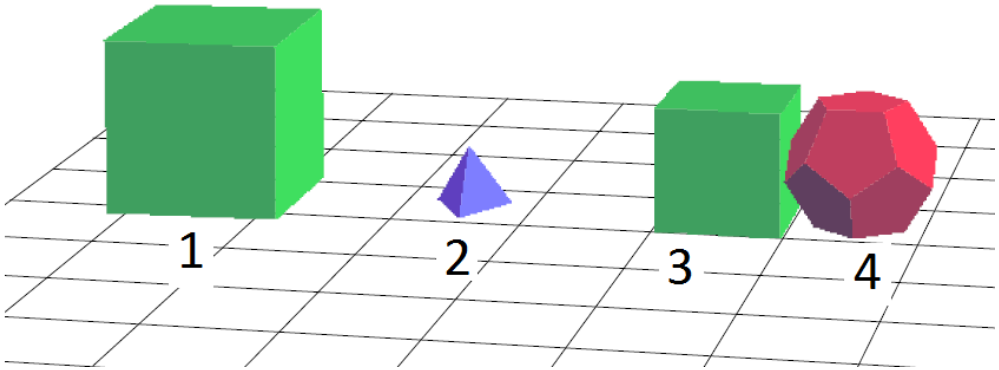
x =	∀x	(Cube(x)	→	Large(x))
1				
2				
3				
4				



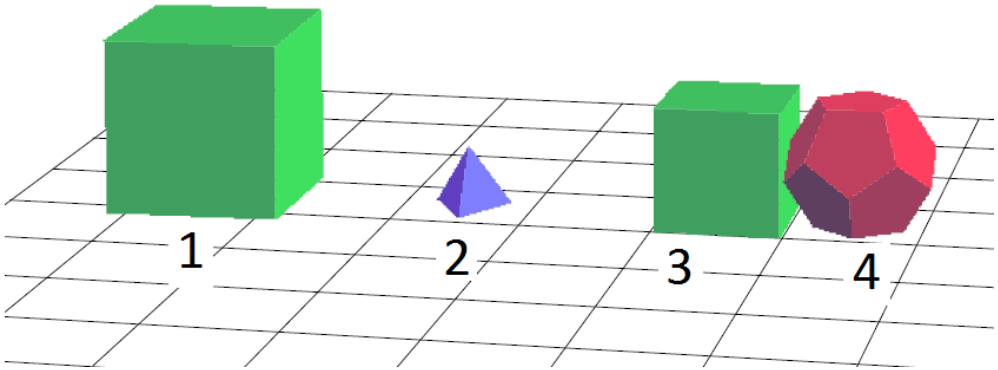
x =	∀x	(Cube(x)	→	Large(x))
1		T		
2		F		
3		T		
4		F		



x =	∀x	(Cube(x)	→	Large(x))
1		T		T
2		F		F
3		T		F
4		F		F

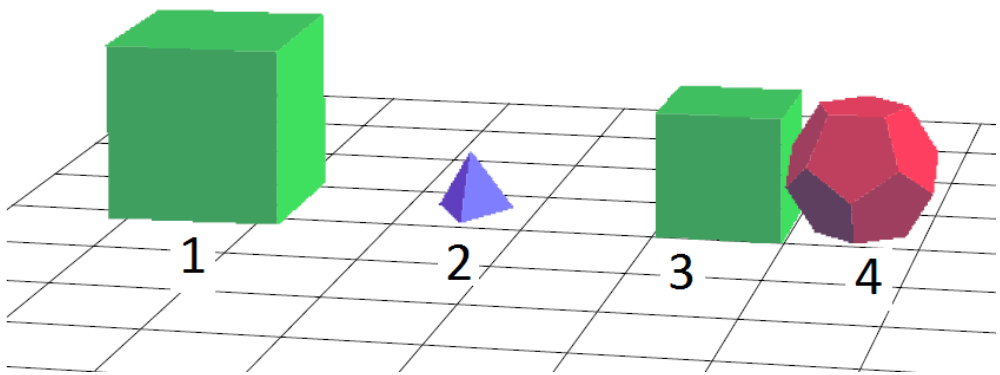


x =	∀x	(Cube(x)	→	Large(x))
1		T	T	T
2		F	T	F
3		T	F	F
4		F	T	F

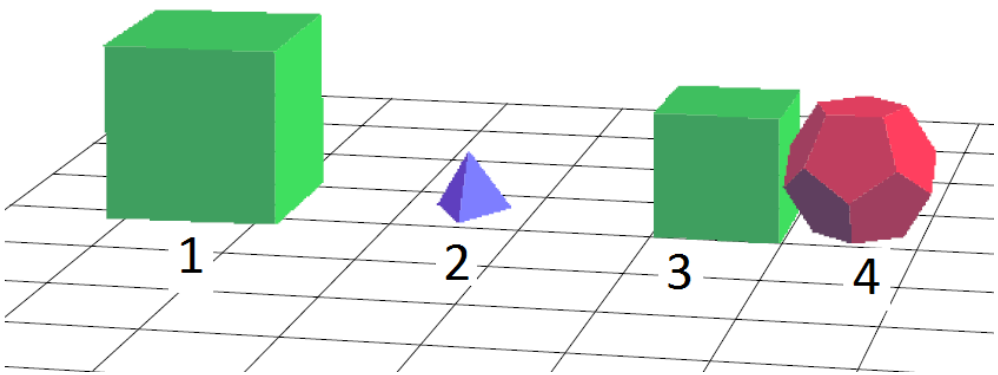


x =	∀x	(Cube(x)	→	Large(x))
1	F	T	T	T
2		F	T	F
3		T	F	F
4		F	T	F

Existential case:

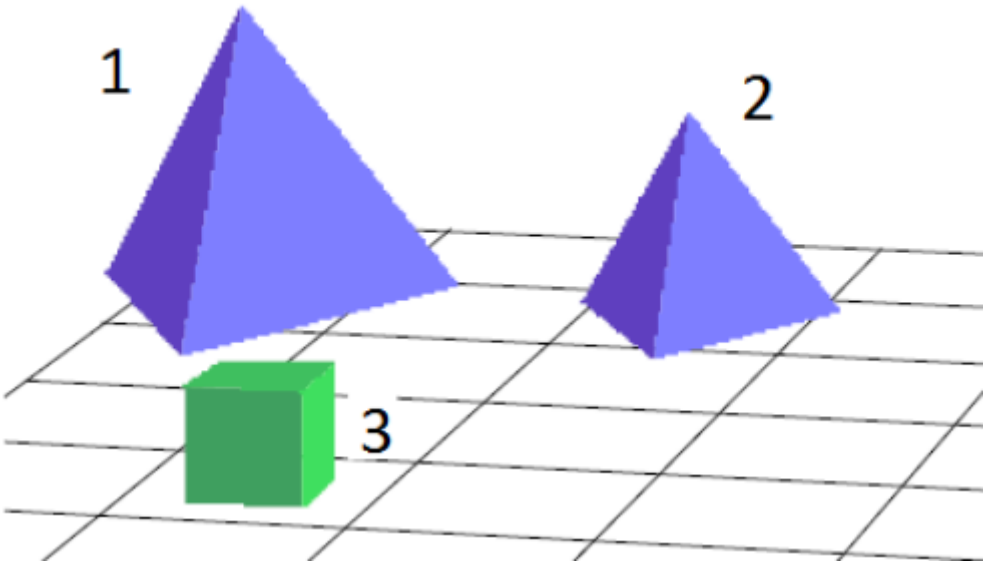


x =	∃x	(Cube(x)	∧	Medium(x))
1		T	F	F
2		F	F	F
3		T	T	T
4		F	F	T



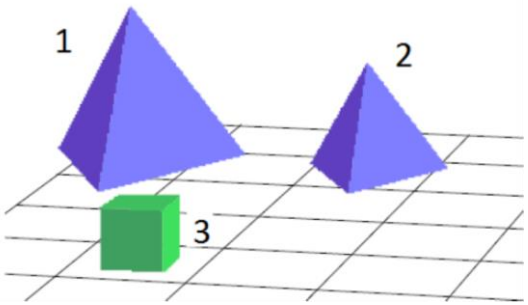
x =	∃x	(Cube(x)	∧	Medium(x))
1	T	T	F	F
2		F	F	F
3		T	T	T
4		F	F	T

Double universal:



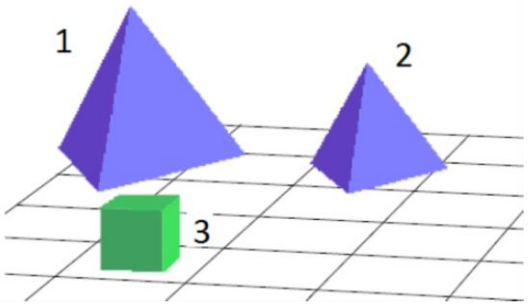
$$\forall x \forall y((\mathrm{Tet}(x) \wedge \mathrm{Cube}(y)) \rightarrow \mathrm{Larger}(x, y))$$

$$\forall x \forall y((\text{Tet}(x) \wedge \text{Cube}(y)) \rightarrow \text{Larger}(x, y))$$



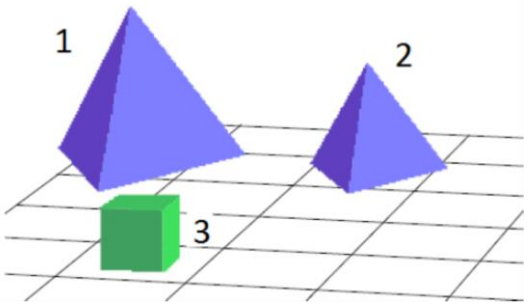
x =	y =	∀x	∀y	((Tet(x)	∧	Cube(y))	→	Larger(x, y))
1	1			T		F		F
	2					F		T
	3					T		T
2	1			T		F		F
	2					F		F
	3					T		T
3	1			F		F		F
	2					F		F
	3					T		F

$$\forall x \forall y((\text{Tet}(x) \wedge \text{Cube}(y)) \rightarrow \text{Larger}(x, y))$$



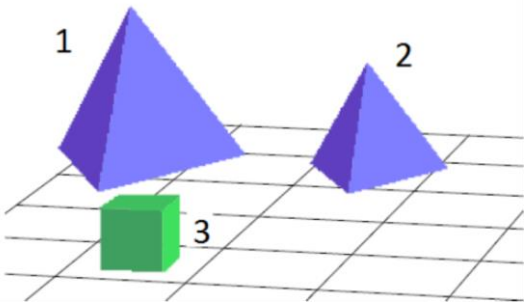
x =	y =	∀x	∀y	((Tet(x)	∧	Cube(y))	→	Larger(x, y))
1	1			T	F	F		F
	2				F	F		T
	3				T	T		T
2	1			T	F	F		F
	2				F	F		F
	3				T	T		T
3	1			F	F	F		F
	2				F	F		F
	3				F	T		F

$$\forall x \forall y((\text{Tet}(x) \wedge \text{Cube}(y)) \rightarrow \text{Larger}(x, y))$$



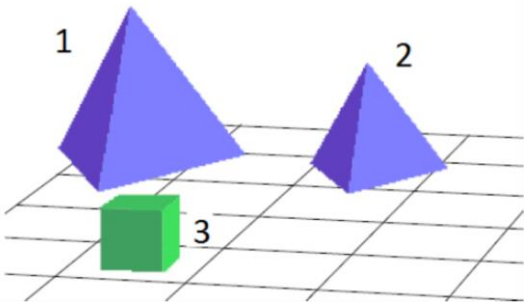
x =	y =	∀x	∀y	((Tet(x)	∧	Cube(y))	→	Larger(x, y))
1	1			T	F	F	T	F
	2				F	F	T	T
	3				T	T	T	T
2	1			T	F	F	T	F
	2				F	F	T	F
	3				T	T	T	T
3	1			F	F	F	T	F
	2				F	F	T	F
	3				F	T	T	F

$\forall x \forall y((Tet(x) \wedge Cube(y)) \rightarrow Larger(x, y))$

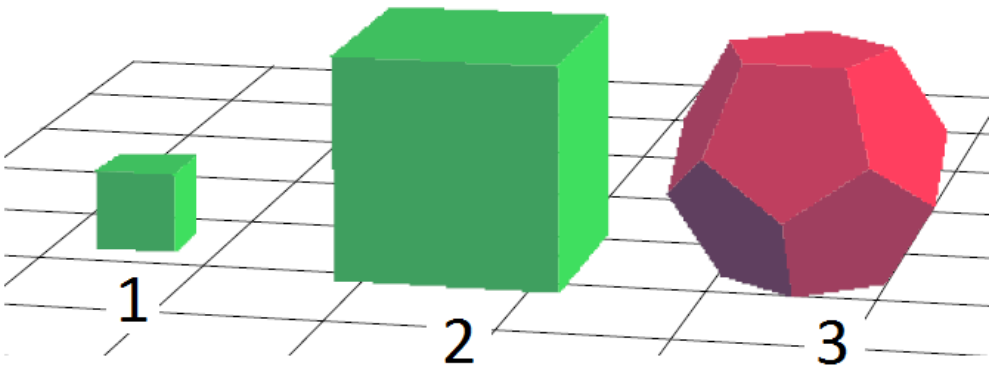


x =	y =	∀x	∀y	((Tet(x)	∧	Cube(y))	→	Larger(x, y))
1	1		T	T	F	F	T	F
	2				F	F	T	T
	3				T	T	T	T
2	1		T	T	F	F	T	F
	2				F	F	T	F
	3				T	T	T	T
3	1		T	F	F	F	T	F
	2				F	F	T	F
	3				F	T	T	F

$\forall x \forall y((Tet(x) \wedge Cube(y)) \rightarrow Larger(x, y))$



x =	y =	∀x	∀y	((Tet(x)	∧	Cube(y))	→	Larger(x, y))
1	1	T	T	T	F	F	T	F
	2				F	F	T	T
	3				T	T	T	T
2	1		T	T	F	F	T	F
	2				F	F	T	F
	3				T	T	T	T
3	1		T	F	F	F	T	F
	2				F	F	T	F
	3				F	T	T	F



x =	y =	∃x	¬	∃y	(Cube(y)	∧	Larger(x, y))
1	1	T	T	F	T	F	F
	2				T	F	F
	3				F	F	F
2	1		F	T	T	T	T
	2				T	F	F
	3				F	F	F
3	1		F	T	T	T	T
	2				T	F	F
	3				F	F	F

Resolving Ambiguity

"Every cube adjoins a tet" is ambiguous.

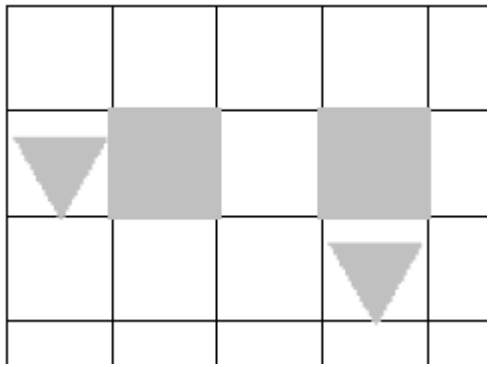
It could mean:

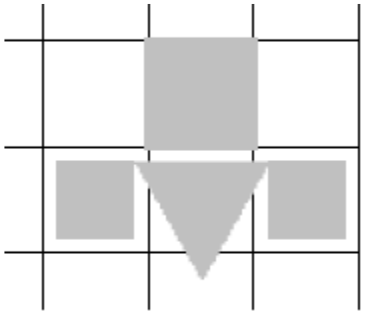
$$\exists \mathbf{y}(\mathbf{Tet}(\mathbf{y}) \wedge \forall \mathbf{x}(\mathbf{Cube}(\mathbf{x}) \rightarrow \mathbf{Adjoins}(\mathbf{x}, \mathbf{y})))$$

Or: $\forall \mathbf{x}(\mathbf{Cube}(\mathbf{x}) \rightarrow \exists \mathbf{y}(\mathbf{Tet}(\mathbf{y}) \wedge \mathbf{Adjoins}(\mathbf{x}, \mathbf{y})))$

What's the difference?

“Bieber” vs. “Mother” readings

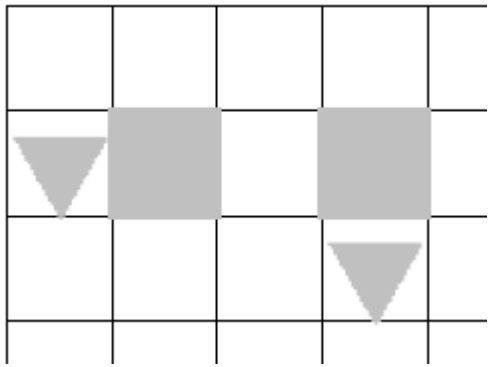


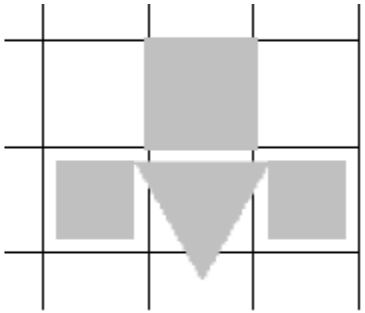


$\exists y(\text{Tet}(y) \wedge \forall x(\text{Cube}(x) \rightarrow \text{Adjoins}(x, y)))$? ?

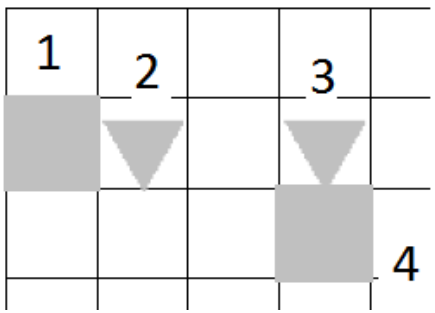
$\forall x(\text{Cube}(x) \rightarrow \exists y(\text{Tet}(y) \wedge \text{Adjoins}(x, y)))$? ?

“Bieber” vs. “Mother” readings

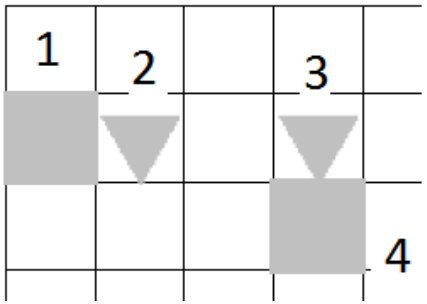




$\exists y(Tet(y) \wedge \forall x(Cube(x) \rightarrow Adjoins(x, y)))$	F	T
$\forall x(Cube(x) \rightarrow \exists y(Tet(y) \wedge Adjoins(x, y)))$	T	T



y =	x =	∃y	(Tet(y)	∧	∀x	(Cube(x))	→	Adjoins(x, y)))
1	1	F	F	F	F	T	F	F
	2					F	T	T
	3					F	T	F
	4					T	F	F
2	1		T	F	F	T	T	T
	2					F	T	F
	3					F	T	F
	4					T	F	F
3	1		T	F	F	T	F	F
	2					F	T	F
	3					F	T	F
	4					T	T	T
4	1		F	F	F	T	F	F
	2					F	T	F
	3					F	T	T
	4					T	F	F



x =	y =	∀x	(Cube(x)	→	∃y	(Tet(y)	∧	Adjoins(x, y)))
1	1	T	T	T	T	F	F	F
	2					T	T	T
	3					T	F	F
	4					F	F	F
2	1		F	T	F	F	F	T
	2					T	F	F
	3					T	F	F
	4					F	F	F
3	1		F	T	F	F	F	F
	2					T	F	F
	3					T	F	F
	4					F	F	T
4	1		T	T	T	F	F	F
	2					T	F	F
	3					T	T	T
	4					F	F	F