Discussion of the paper

Why the Observed Motion History of World Trade Center Towers is Smooth

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DOI: 10.1061/_ASCE_EM.1943-7889.0000198

Journal of Engineering Mechanics, Vol. 137, No. 1, January 1, 2011, pg. 82-84

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1. Introduction

In their paper, Le and Bažant respond to the claim that the motion of the roofline of WTC 1, as captured in video footage, is inconsistent with the hypothesis of gravity-driven progressive collapse. Unfortunately they do not give any sources for this claim, but it is likely that they are responding to the work of Chandler (2010) and MacQueen and Szamboti (2009).

It is agreed on all sides that the collapse of WTC 1 initiated at the 98th floor leaving a 12-story upper part to fall onto a stationary 97-story lower part, as stated by NIST NCSTAR 1-6, p. 156. Le and Bažant calculate the total velocity reduction after impact to be about 3%. They also find that, after impact, the upper part continues to accelerate downwards at 6.2 m/s^2. It seems these calculations are based on assumptions, especially regarding the steel columns on story 97, which are without justification and contradicted by NIST.

2. Inertia Resistance

Le and Bažant first calculate the slowing of the upper portion due to the inertia of the first story impacted. For reasons that are not specified, they consider only the mass of the concrete floor slab to be involved in this exchange of momentum. They calculate the effect of a descending mass of 54.18 Mkg striking a stationary mass of 0.627 Mkg. However, the concrete floor slab is only part of the overall floor mass, which also includes rebar, steel decking, trusswork, and the live load. According to Bažant and Le (2008, p. 905), from which Le and Bažant obtain the data used, m_2 = the mass of a single story is 3.87 Mkg for WTC 1. Using this value, we get a velocity ratio of 54.18/(54.18 + 3.87) = 0.93. The velocity lost is therefore about 7% of the original, rather than the 1.1% claimed. (Note that this is already more than the 3% *total* loss, calculated by Le and Bažant.)

3. Column resistance

For simplicity, Le and Bažant's calculations assume that the 287 columns on the 97th story are identical. Unfortunately, the full specifications of this representative column are not stated. We are told that the plastic moment M_p for this column is 0.32 MNm, and from Equation (3) we can infer that the yield stress σ_0 = 250 MPa. The total cross-sectional area of the 287 columns is stated to be 6.05 m^2. The shape of the column, its overall dimensions, and flange and web thicknesses are not given. We can find no specification consistent with this data.

Most of the columns (240 of the 287) were perimeter columns, the overall dimensions and shape of which are stated by NIST (NCSTAR 1-3D, p. 4) to be approximately 14" square box columns, i.e. having width and breadth equal to 0.3556 m. To calculate M_p we used a standard formula for the plastic section modulus of a hollow rectangular section (see Gaylord et al, 1979, 7-3), putting width equal to breadth *b*, web thickness equal to flange thickness *t*, and multiplying by the yield stress, gives:

$$Mp = \frac{b^3}{4}\left(1-\left(1-\frac{2t}{b}\right)^3\right)\sigma_0 \qquad (1)$$

Calculating backwards (from M_p=0.32 MNm) gives t = 7.02 mm. This is much less than the 10 mm thickness given in Bažant and Le (2008, p. 896) for the aircraft impact level, and even a little less than the 7.5 mm they state for the top story. It also entails a total cross-sectional area of 287 x 4 x 0.3556 x 0.00702 = 2.87 m^2, which is less than half of the 6.05 m^2 stated. The authors need to explain how their M_p value was obtained.

Our estimate of the average plastic moment of the columns on story 97 is 0.64 MNm, obtained as follows. For the perimeter columns, we conservatively assume web and flange thicknesses t = 7.5 mm. The yield stress of the perimeter columns at story 97 is reported by NIST to be 55ksi – 100ksi (NCSTAR 1-6, p. 61, and NCSTAR 1-3B, Table 4-2, p. 52). We estimate the average yield stress to be 65ksi, i.e. 450 MPa, which is also conservative, since NIST reports the measured yield stresses to be above nominal. (NCSTAR 1-6, p. 61). This gives M_p = 0.61 MNm for the perimeter columns.

The core columns vary in size and steel types. They are wide-flange columns, with flanges ranging from 16.695" x 3.033" down to 8" x 0.528", and either 36, 42, 45, or 50 ksi yield strength. (See the available NIST SAP2000 model data, reproduced by MacQueen and Szamboti (2009), pp. 22-3.) To calculate M_p for the weak axis, the plastic section modulus $Z_p = ½\, t.b^2$, also obtained from Gaylord et al (1972, 7-3), was used, omitting the small contribution from the web. The M_p values for core columns were found to range from 2.01 MNm to 0.09 MNm, the average being 0.75 MNm. The weighted average, for core and perimeter columns, is then 0.64 MNm. We conclude that 0.32 MNm is much too low.

Using this corrected M_p value, together with the other column data stated above, we can repeat Le and Bažant's calculations for the velocity reduction of the upper part of WTC 1. First we calculate the total yield load for all columns. For the 240 perimeter columns: $P = 240 \times 4bt\sigma_0 = 1{,}150$ MN. For the core, using the NIST data, the total cross-sectional area of the core columns is found to be 1.69 m^2, and maximum load is 460 MN. In total, we have $P = 1{,}610$ MN.

For calculating the load-displacement curve we also need the column length L, given by Le and Bažant as 3.7 m for all the columns. Bažant and Zhou (2002, p. 5) state the effective height of the perimeter columns to be 2.5 m, the distance between the 1.32 m deep spandrel plates, that were heavier gauge than the adjacent column webs. (See NIST NCSTAR 1-3A, pp. 7-9.) Since our aim is to calculate a conservative estimate of the velocity drop, however, we will ignore the spandrel plates and use $L = 3.7$ m – which makes the perimeter columns more slender, substantially reducing their resistance during buckling. The resistive force F_b is then given by the formula below (see Bažant and Zhou 2002, p. 6) where number of columns is n, and u the reduction in column length.

$$F_b = \frac{4\,n\,M\,p}{L\sqrt{1-\left[1-\left(\frac{u}{L}\right)^2\right]}} \qquad (2)$$

Using Mp = 0.64 MNm we get the graph shown in Fig. 1.

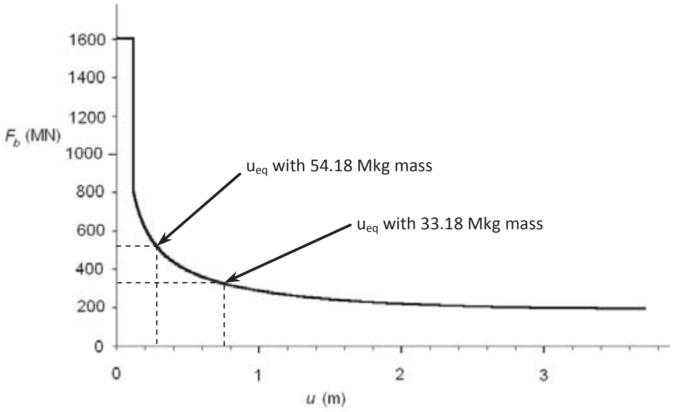

Fig. 1. Diagram of load vs. displacement during axial deformation and buckling

The average resistance of the columns is 310 MN, using numerical integration. The displacement u_{eq}, at which column resistance equals the 530 MN weight of the upper part (i.e. the 54.18 Mkg mass used by Le and Bažant) is 0.27 m, rather than the 0.065 m claimed.

Up to this point we have used Le and Bažant's mass value of 54.18 Mkg for the upper part of the tower, but this conflicts with the NIST report (NCSTAR 1-6D, p. 176, Table 4-7), which states the actual total load on the columns between floors 98 and 99 to be 73,143 kips, i.e. 325.4 MN or 33.18 Mkg. NIST's estimate is also much closer to typical mass per square meter values for other buildings sharing this type of construction, such as the Sears (now Willis) Tower and John Hancock building. For a detailed examination of the masses of WTC 1 and 2 see Urich (2007).

From here on, we will use NIST's 33 Mkg figure in our calculations. For example, u_{eq} then occurs at roughly 0.76 m, as shown in Fig. 1.

4. Calculating the Velocity Curve

To verify the accuracy of the gravity-driven model, we can calculate the velocity curve for the roof line, and compare it with the behavior of WTC 1 itself. Fortunately, there is high-resolution footage of the collapse of WTC 1 shot by professional filmmaker Etienne Sauret, and used for the documentary film *WTC - The First 24 Hours* (2002). Each pixel of this footage represents 0.27 m of the tower, and frame rate is 30 per second, allowing for accurate measurements of the motion.

David Chandler has analyzed this motion using Tracker, an open source video analysis tool. His graph is shown below, together with a calculated velocity plot for a gravity-driven collapse.

The calculated velocity of the roofline was obtained numerically using the load-displacement curve shown above, and scaling up linearly for lower stories, according to the increasing design load. We also assumed Le and Bažant's freefall acceleration during the collapse of the first story. Floors are treated as rigid and incompressible, and assumed to stick together upon impact. The upper part of the building is modeled as a rigid block, which Le and Bažant regard as a reasonable approximation.

It is easy to derive an approximation of this curve, using hand calculations, given the average 97th story column resistance of 310 MN, which is approximately NIST's (325.4 MN) weight for the upper part of the building. Hence the average velocity is approximately constant after the first impact – decreasing slightly due to the inertia of the impacted stationary floors.

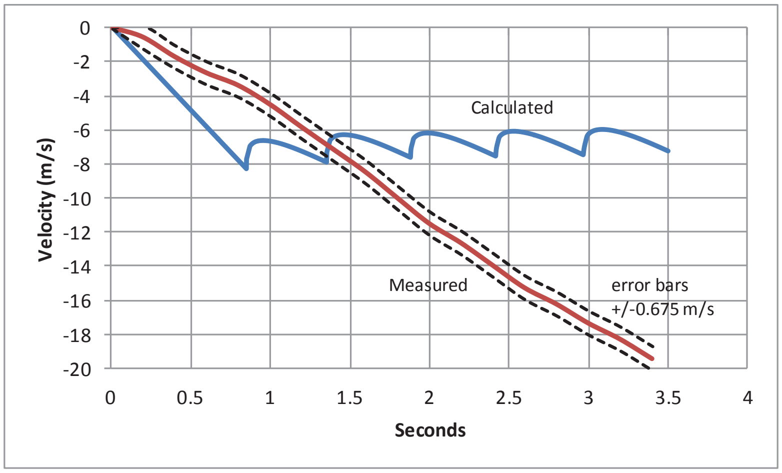

Fig. 2. Measured and calculated velocity curves

The calculated first velocity decrease is 1.65 m/s (approximately 20%), and would be visible (if it existed) in a velocity plot obtained from the Sauret video footage. Also, the predicted average acceleration after impact (roughly zero) is significantly different from what was observed.

5. Conclusion

The analysis of Le and Bažant uses incorrect input values. These errors each have the effect of reducing the resistance of the lower part of the building. As a result, their calculated velocity drop on impact is too low, and their calculated acceleration following that drop is too high.

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