First-Order Goggles

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Fancier than Boolean goggles!

How they work

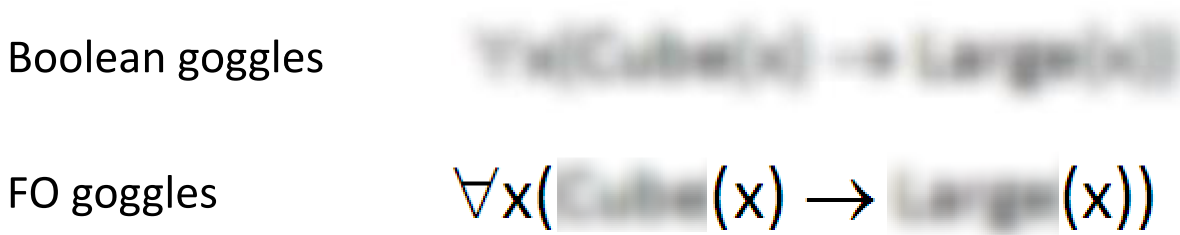
- FO goggles are used for looking at FOL sentences. Some parts of the sentence are clearly visible through the goggles, but other parts become fuzzy and illegible.

- The Boolean sentential operators $\wedge$, $\vee$, $\neg$, $\rightarrow$ and $\leftrightarrow$ are clearly visible through the goggles.
- Brackets ‘(‘ and ‘)’ are also visible.
- The operators ‘$\forall x$’, ‘$\exists x$’ etc. are visible, including the variables.
- The identity predicate ‘=’ is visible. **All other predicates are fuzzy.**

Boolean goggles vs. FO goggles

- The FO goggles are much better than Boolean goggles, in the sense that you see much more sentence structure through them.

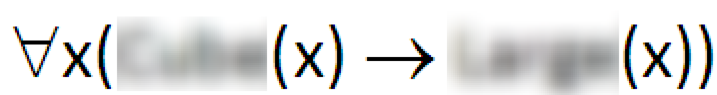
- E.g. $\forall x(\mathrm{Cube}(x) \to \mathrm{Large}(x))$ becomes:



- Note that the Boolean goggles cannot see *anything* that is inside the scope of a quantifier. Even Boolean operators are invisible, when inside the scope of a quantifier.

- $\forall x(\text{Cube}(x) \rightarrow \text{Large}(x))$ is
  -- just 'P', through Boolean goggles.

- Using FO goggles we can write using 'nonsense' (meaningless)
  $\forall x(\text{Cobo}(x) \rightarrow \text{Lenga}(x))$, or
  $\forall x(P(x) \rightarrow Q(x))$.



Quiz

- Write the following sentences, as they appear through:
  i. Boolean goggles
  ii. FO goggles

1. $\forall y\ \mathrm{Cube}(y) \rightarrow \exists z\ \mathrm{Small}(z)$

BG: $P \rightarrow Q$

FOG: $\forall y\ \mathrm{Crub}(y) \rightarrow \exists z\ \mathrm{Sinill}(z)$

2. $\neg\exists x\ Cube(x)$

BG: $\neg P$

FOG: $\neg\exists x\ P(x)$

3. $a = b$

BG: P

FOG: $a = b$

4. $\forall x\ (Tet(x) \rightarrow SameRow(c, x)) \vee Small(c)$

BG: $P \vee Q$

FOG: $\forall x\ (P(x) \rightarrow Q(c, x)) \vee R(c)$

5. $\forall x\, Cube(x) \rightarrow \neg\exists x\, \neg Cube(x)$

BG: $P \rightarrow \neg Q$

FOG: $\forall x\, P(x) \rightarrow \neg\exists x\, \neg P(x)$

6. $[\forall z\ (Cube(z) \rightarrow Large(z)) \wedge Cube(b)] \rightarrow Large(b)$

BG: $[P \wedge Q] \rightarrow R$

FOG: $[\forall z\ (P(z) \rightarrow Q(z)) \wedge P(b)] \rightarrow Q(b)$

7. $\exists x\, (Cube(x) \wedge x \neq a)$

BG: P

FOG: $\exists x\, (P(x) \wedge x \neq a)$

TT con vs. FO con

- You remember that the argument below isn't TT con?

  Cube(a)

  a = b

  Cube(b)

- Wasn't that disappointing?

- Now put the FO goggles on. How does it look?

  Criba(a)

  a = b

  Criba(b)

- Can you see that the argument is a logical consequence?
- Yes you can!
  - So the argument is ‘FO consequence’.

FO equivalent

- $\exists x(\text{Small}(x) \wedge x = c) \Leftrightarrow \text{Small}(c)$?

- Yes. Are they also “FO equivalent”? Put the FO goggles on.

- $\exists x(P(x) \wedge x = c) \Leftrightarrow P(c)$