



Looking into the future

(of your proof)

Unbreakable premises

- Many proofs follow the ‘renovation procedure’ of demolition, followed by construction.

(You break the premises into small sentences, using elimination rules, then put the pieces together in a different pattern.)

- But some sentences are ‘unbreakable’. There is no elimination rule that will take them apart.

Unbreakable premises

- E.g. Sentences of these forms are unbreakable:

$$\neg(P \wedge Q)$$

$$\neg(P \vee Q)$$

$$\neg(P \rightarrow Q)$$

$$\neg(P \leftrightarrow Q)$$

- So when you see an unbreakable sentence in the premises of an argument, what do you do?



Crystal Balls

- In fact there's (usually) only one thing you can do with an unbreakable sentence, and that's to use it as a premise with \perp Intro (to introduce \perp of course).
- Since this is the only thing you can do, an unbreakable sentence can be viewed as a 'crystal ball' that allows you to glimpse some things in the future of your proof.

- E.g. the sentence $\neg(\text{Cube}(a) \wedge \text{Tet}(b))$ can only be used to introduce \perp .
- So your proof will (sooner or later) include the lines:

$\text{Cube}(a) \wedge \text{Tet}(b)$

\perp

- In other words, the sentence $\neg(\text{Cube}(a) \wedge \text{Tet}(b))$ shows you:

