Quantifier Equivalences (these cannot be used on formal proofs in F)

(Memorise as many as you can. The importance decreases as you go down the page.)

Quantifier de Morgan

$\neg\forall x\ P(x) \Leftrightarrow \exists x\ \neg P(x)$
$\neg\exists x\ P(x) \Leftrightarrow \forall x\ \neg P(x)$

Distribution Laws

$\forall x\ (P(x) \wedge Q(x)) \Leftrightarrow \forall xP(x) \wedge \forall xQ(x)$
$\exists x\ (P(x) \vee Q(x)) \Leftrightarrow \exists xP(x) \vee \exists xQ(x)$

Warning !! $\forall x\ (P(x) \vee Q(x)) \not\Leftrightarrow \forall xP(x) \vee \forall xQ(x)$ $[\forall x\ (P(x) \vee Q(x)) \not\Rightarrow \forall xP(x) \vee \forall xQ(x)]$
$\exists x\ (P(x) \wedge Q(x)) \not\Leftrightarrow \exists xP(x) \wedge \exists xQ(x)$ $[\exists x\ (P(x) \wedge Q(x)) \not\Leftarrow \exists xP(x) \wedge \exists xQ(x)]$

Quantifying over a sentence, or a wff with a different variable

$\forall x\ P \Leftrightarrow P$
$\exists x\ P \Leftrightarrow P$

$\forall x\ (P \vee Q(x)) \Leftrightarrow P \vee \forall xQ(x)$
$\exists x\ (P \wedge Q(x)) \Leftrightarrow P \wedge \exists xQ(x)$

Quantifying over conditionals

$P \rightarrow \forall xQ(x) \Leftrightarrow \forall x(P \rightarrow Q(x))$
$P \rightarrow \exists xQ(x) \Leftrightarrow \exists x(P \rightarrow Q(x))$ $\exists x(P \rightarrow Q(x))$ is an ok sentence, even though $\exists x(P(x) \rightarrow Q(x))$ isn't.

$\exists xQ(x) \rightarrow P \Leftrightarrow \forall x(Q(x) \rightarrow P)$ "If anything is Q, then P"
$\forall xQ(x) \rightarrow P \Leftrightarrow \exists x(Q(x) \rightarrow P)$ *Yes, this one does look really weird.*