Material and Spiritual Conditionals

Richard Johns
Department of Philosophy
Langara College

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1. What's Up With That?

One would think that logic, as a subject, would be logical. It should make sense. Unfortunately this is not the experience of many students of logic. They find certain logical claims hard to swallow.

There are two main such claims. One is the logical principle *ex falso quodlibet*, or "from a contradiction, anything follows". (I won't say anything more about that principle here.) The other is the way that *conditionals* are analysed in sentential logic. In short, "If A then B" is understood to mean "either A is false, B is true, or both", which is symbolised as $\neg A \vee B$, or equivalently $A \rightarrow B$.

This analysis of the (plain, indicative) conditional "If P then Q" just seems plain wrong. After all, its assertion doesn't require that there be any *logical connection* between A and B. It has nothing to do with (for example) being able to infer B from A. Whereas "If A then B" seems to express that A leads logically to B, or something like that.

This fact that no connection between A and B needs to exist, for the conditional $A \rightarrow B$ to hold, means that some surprising inferences are valid. For example:

$$\begin{array}{l} \quad B \\ \hline \therefore A \rightarrow B \end{array}$$

And even worse:

$$\begin{array}{l} \quad \neg A \\ \hline \therefore A \rightarrow B \end{array}$$

(These are easily verified, using a truth table, to be TT consequences.)

How is one to deal with this (unfortunate) fact, that students don't like the way logicians analyse the conditional? There are three main approaches.

(I) Blame the conditional

Barwise and Etchemendy, for example, dismiss non-truth functional connectives (such as conditionals) by saying that "their meanings tend to be vague and subject to conflicting interpretations" (*Language Proof and Logic*, p. 177). I see. The poor conditional is such a wretched mess that we simply can't have it in our (nice, clean) language! We have to put something more presentable there in instead.

I must say that this seems awfully procrustean. The fact is that our semantics, based on the idea that the meaning of a sentence is its truth conditions, is inadequate to handle conditionals. So like Procrustes, rather than get a bigger bed we instead hack the legs off the conditional.

(II) Blame the student

A second approach is to suggest that students who see something inadequate in the standard analysis just don't get it, and are perhaps unsuited to the study of logic. They are unable to ascend to the required level of abstraction, perhaps. Unfortunately I was myself one of these students, and apparently I still am. So this approach, though attractive, is unavailable to me.

(III) Blame the system

As mentioned in approach (I), FOL is based on the idea that the meaning of a sentence is its truth conditions. Perhaps this idea is not only false (which it surely is) but also inadequate for the understanding of conditionals? In that case, any "conditional" that appeared in FOL could, at best, be a shadow of the real thing. This is the approach I take. Now, it is rather helpful that the conditional $A \rightarrow B$ in FOL is called the *material* conditional, for this helps us to distinguish it from the *real* conditional, "If A then B". Indeed, it's rather tempting to call the real conditional "spiritual", by contrast. I shall yield to this temptation, as it rather fits with Frank Ramsey's analysis of the (real) conditional, in terms of belief dynamics (*spirituel* = of or relating to the mind or intellect).

2. Believing a conditional

We have seen that sentences are uttered to express beliefs. Thus a conditional sentence must also express something about one's epistemic state. But what? After all, when Betty says "if *A* then *B*", she is usually unsure about the truth of both *A* and *B*. (The answer I give here is essentially the one given by Frank Ramsey in 1931, and endorsed by others since then.)

Suppose Betty believes this sentence (If *A* then *B*) when in the epistemic state *K*, even though she believes neither *A* nor *B*. Her belief in "if *A* then *B*" consists (according to Ramsey) in the fact that *B* is believed in the hypothetical state K+A, where the information that *A* is added to *K*. I.e.

Definition "If A then B" is believed in K iff B is believed in K+A.

For example, suppose K is a state where it is known that all fish swim, but it is not known whether or not Sammy is a fish, or whether Sammy swims. In the state K + (Sammy is a fish), however, it *is* known that Sammy swims, since this is easily inferred. Hence, according to the above definition, the conditional "If Sammy is a fish then Sammy swims" is believed in K.

In general, therefore, one believes (and will assert) a conditional "If A then B" in cases where one possesses information which, when A is added to it, allows one to conclude B.

Note that, in a conditional "If A then B", A is called the *antecedent* and B the *consequent*.

3. Probable Conditionals

What if B isn't *certain* in the state K+A but merely *probable*? In such cases the conditional itself is merely probable, and will be expressed tentatively in some form such as: "It is probable that if A then B", or "if A then probably B".

Using the notation that $P_K(A)$ is the (epistemic) probability of A in the epistemic state K, we have:

Corollary $P_K(\text{If } A \text{ then } B) = P_{K+A}(B) = P_K(B \mid A)$.

In other words, the probability of a conditional is a conditional probability, namely the probability of the consequence on the antecedent.

For example, I have a loonie in my hand. I don't think I'll toss it ten times in a row. But suppose I did? In that hypothetical state I believe that I'll get at least one head. But I'm not certain of this. It's probable that, if I toss the coin 10 times, then I'll get at least one head.

4. Conditionals and Logical Consequence

A person who says "If A then B" is basically inferring B from A. So then, "If A then B" and "B is a logical consequence of A" are the same thing, right? Not quite. There are a couple of differences.

First, a person who says "If A then B" isn't claiming that A *by itself* entails B, since usually they use "background information" in the inference.

Second, the claim "if A then B" isn't *quite* the same as a claim that B is a consequence of A, in conjunction with my background knowledge. There is a difference between *asserting* that you believe P and *expressing* a belief that P. For example, saying "Sammy is a fish" certainly *expresses* a belief that Sammy is a fish, but it doesn't explicitly *claim* that I believe that Sammy is a fish. After all, "Sammy is a fish" is a biological claim about Sammy, whereas "I believe Sammy is a fish" is a psychological claim about me. In a similar way, asserting a conditional "if A then B" *expresses* a consequence, but does not assert one.

5. The Case of the Irrelevant Antecedent

In the usual cases where "if A then B" is asserted, in state *K*, one believes neither A nor B in *K*. But what about other cases? What if, for example, B is believed in both *K* and *K+A*? Since *B* is believed in *K+A*, our definition is satisfied, so that "If *A* then *B*" holds in *K*. Yet *A* is *irrelevant* here. We're not deriving *B* from *A*, since *B* is there already.

Is the definition correct? Do we assert conditionals with such irrelevant antecedents, such as:

> "If this coin lands heads then it won't turn into a chicken" ??

Well, we don't say such things very often. But we do say them sometimes, such as when trying to *trick* people without actually lying.

For example, Fred's mom hands him $5, telling him to give it to his brother Mike. Fred will certainly obey his mother in such a matter. So Fred runs into Mike later, and says:

"Hey Mike. You see this crisp five dollar bill? I'll give it to you *if* you tidy my room for me"

As he says this, Fred knows that he will give Mike the bill either way, whether the room is tidied or not, to obey his mother's instruction. But he's counting on the fact that Mike doesn't know this. Mike will, perhaps, tidy the room in order to secure the money. Surely what Fred says here is *true*. Moreover, Fred *believes* it.

The conditional is deceptive, it seems, due to the conversational rule of *Say the strongest relevant thing you believe*. Fred breaks this rule, since he believes "I will give Mike the bill", which is stronger than "If Mike cleans my room then I will give Mike the bill", yet he asserts only that weaker claim. Fred thus suggests, by conversational implication, that there's some doubt about whether Mike will get the money, if he doesn't clean Fred's room. This is false.

So Ramsey's analysis handles conditionals with irrelevant antecedents quite well. Basically it's quite true that, if this loonie here lands heads, then it won't turn into a chicken. I believe it with all my heart.

The situation where *B* holds in both *K* and *K+A* can be expressed more honestly by saying "Even if *A*, *B*."

6. What about a *semantics?*

Philosophers who have learned that meaning is truth conditions will be waiting for me to give a semantics for the conditional. After all, so far I've just given Ramsey's "belief test" for the conditional. What are the truth conditions, eh?

The meaning of a conditional cannot be given in terms of truth conditions. You have to give *belief conditions* instead. And I've already done that. Ramsey's principle isn't a mere "belief test", it's a semantics for the conditional. And the correct semantics too.

7. Rules of inference for conditionals

What rules of inference are sound for conditionals, according to the Ramsey semantics? Is modus ponens sound, for example?

We can easily see that modus ponens (i.e. conditional elimination) is sound.

If A then B
A

∴B

For suppose that the premises both hold (with certainty) in some epistemic state K. According to the Ramsey semantics, this means that B holds in K+A, and A holds in K. But if A holds in K, then K+A = K. It then follows that B holds in K. Hence there's no epistemic state in which the premises all hold, but not the conclusion.

Similarly, conditional proof (conditional introduction) is sound.

A

∴B

∴If A then B

Let K be the epistemic state generated by full belief in the premises. Then, since the expansion of K by A allows B to be proved, it follows that B holds in K+A. But then, by the Ramsey semantics, "If A then B" holds in K.

8. Those *Funny* Inferences

What does the Ramsey semantics have to say about the "funny inferences" that hold for the material conditional? First let's consider:

$$\begin{array}{l} \quad B \\ \text{----------} \\ \therefore A \rightarrow B \end{array}$$

Given that B holds in K, does B hold in K+A? Well, actually it depends on what you mean by "hold". Deductive logic is the logic of *certainty*. A deductive inference is one where the conclusion is certain, given that the premises are certain. So let's suppose that B is *certain* in K. Now, one feature of certainty, or dogmatism, is that it resists updating in the light of new information. So even in cases where A undermines B, B will still be certain in K+A.

So the Ramsey semantics actually *endorses* this argument form, at least in the deductive case. In general, however, this is not a reasonable inference. If B is anything less than perfectly certain in K, then it might have low probability in K+A. $P_K(B)$ can be high, and $P_K(B \mid A)$ very low. This argument form does not preserve epistemic probability, therefore. No wonder it seems fishy!

Now let's look at the other (even worse) one.

$$\begin{array}{l} \quad \neg A \\ \text{-----------} \\ \therefore A \rightarrow B \end{array}$$

Suppose $\neg$A holds (for certain) in K. Does B hold in K+A? Well, if $\neg$A holds in K, then K+A is an impossible (contradictory) epistemic state. I'm inclined to say that K+A does not even exist. So, again, it's no wonder that this inference seems fishy.

In the general (non-deductive) case, $\neg$A is merely probable in K. So K+A isn't a contradictory state. And in general, some other proposition B won't be probable in K+A, so the inference is ridiculous. It's basically arguing that since P(A) is low, P(B | A) is high!

For this inference to make sense, there are two requirements:

(i) You must be doing deductive logic, i.e. the premise $\neg$A is known with certainty.

(ii) You must accept the existence of contradictory epistemic states, and the principle *ex falso quodlibet* (anything follows from a contradiction).

So, the Ramsey semantics helps us to understand why this inference seems even worse than the first. It requires an additional dicey assumption.

9. How are the material and spiritual conditionals related?

We have seen that rules of inference *conditional introduction* and *conditional elimination* both hold for the spiritual conditional, and it's easy enough to see that they also hold for the material conditional. Using this fact one can give an (apparent) proof of their equivalence.

For convenience, let's use a spooky arrow ~> to represent the spiritual conditional.

1.	$A \mathbin{\sim>} B$	
2.	$\quad A$	
3.	$\quad B$	$\sim>$ Elim: 1, 2
4.	$A \rightarrow B$	$\rightarrow$ Intro: 2-3

1.	$A \rightarrow B$	
2.	$\quad A$	
3.	$\quad B$	$\rightarrow$ Elim: 1, 2
4.	$A \mathbin{\sim>} B$	$\sim>$ Intro: 2-3

There must be something wrong here, however, since the material and spiritual conditionals are not equivalent. For example, we have seen that $P_K(A \mathbin{\sim>} B) = P_K(B \mid A)$, whereas it's easy to see that $P_K(A \rightarrow B) \neq P_K(B \mid A)$. After all, $P_K(A \rightarrow B) = P_K(\neg A \vee B) \geq P_K(B)$. Yet $P_K(B \mid A)$ can be less than $P_K(B)$. Hence $P_K(A \mathbin{\sim>} B) \neq P_K(A \rightarrow B)$, so that the two conditionals are not equivalent.

Let's look at how the two arguments do on probability preservation. The first one is fine. For let's assume that $P(A \mathbin{\sim>} B)$ is high, so that $P(B \mid A)$ is high. Then $P(\neg B \mid A)$ is low. Now $P(A \rightarrow B) = P(\neg(A \wedge \neg B)) = 1 - P(A \wedge \neg B) = 1 - P(A)P(\neg B \mid A)$. This is high, as required.

The second argument is a different story. If $P(B)$ is high and $P(B \mid A)$ is low, for example (as is perfectly possible) then the premise will be far more probable than the conclusion. Hence the inference is very weak, even though it is deductively valid. We see that, where spiritual conditionals are concerned, mutual *deductive* consequence is insufficient for logical equivalence.