Rules of Inference in $\mathcal{F}^{+}$

Conjunction Introduction
($\wedge$ Intro)

$$\begin{array}{l|l} & P_1 \\ & \Downarrow \\ & P_n \\ & \vdots \\ \triangleright & P_1 \wedge \ldots \wedge P_n \end{array}$$

Conjunction Elimination
($\wedge$ Elim)

$$\begin{array}{l|l} & P_1 \wedge \ldots \wedge P_i \wedge \ldots \wedge P_n \\ & \vdots \\ \triangleright & P_i \end{array}$$

Disjunction Introduction
($\vee$ Intro)

$$\begin{array}{l|l} & P_i \\ & \vdots \\ \triangleright & P_1 \vee \ldots \vee P_i \vee \ldots \vee P_n \end{array}$$

Disjunction Elimination
($\vee$ Elim)

$$\begin{array}{l|l} & P_1 \vee \ldots \vee P_n \\ & \vdots \\ & \quad | \; P_1 \\ & \quad | \; \vdots \\ & \quad | \; S \\ & \Downarrow \\ & \quad | \; P_n \\ & \quad | \; \vdots \\ & \quad | \; S \\ & \vdots \\ \triangleright & S \end{array}$$

Disjunctive Syllogism
(DS)

$$\begin{array}{l|l} & P \vee Q \\ & \vdots \\ & \neg P \\ & \vdots \\ \triangleright & Q \end{array} \qquad \begin{array}{l|l} & P \vee Q \\ & \vdots \\ & \neg Q \\ & \vdots \\ \triangleright & P \end{array}$$

Negation Introduction
($\neg$ Intro)

$$\begin{array}{l|l} & \quad | \; P \\ & \quad | \; \vdots \\ & \quad | \; \bot \\ \triangleright & \neg P \end{array}$$

Negation Elimination
($\neg$ Elim)

$$\begin{array}{l|l} & \neg\neg P \\ & \vdots \\ \triangleright & P \end{array}$$

$\bot$ Introduction
($\bot$ Intro)

$$\begin{array}{l|l} & P \\ & \vdots \\ & \neg P \\ & \vdots \\ \triangleright & \bot \end{array}$$

$\bot$ Elimination
($\bot$ Elim)

$$\begin{array}{l|l} & \bot \\ & \vdots \\ \triangleright & P \end{array}$$

Conditional Introduction
($\rightarrow$ Intro)

$$\begin{array}{l|l} & \quad | \; P \\ & \quad | \; \vdots \\ & \quad | \; Q \\ \triangleright & P \rightarrow Q \end{array}$$

Conditional Elimination
($\rightarrow$ Elim)

$$\begin{array}{l|l} & P \rightarrow Q \\ & \vdots \\ & P \\ & \vdots \\ \triangleright & Q \end{array}$$

Biconditional Introduction
($\leftrightarrow$ Intro)

$$\begin{array}{l|l} & \quad | \; P \\ & \quad | \; \vdots \\ & \quad | \; Q \\ & \quad | \; Q \\ & \quad | \; \vdots \\ & \quad | \; P \\ \triangleright & P \leftrightarrow Q \end{array}$$

Biconditional Elimination
($\leftrightarrow$ Elim)

$$\begin{array}{l|l} & P \leftrightarrow Q \text{ (or } Q \leftrightarrow P) \\ & \vdots \\ & P \\ & \vdots \\ \triangleright & Q \end{array}$$

Reiteration
(Reit)

$$\begin{array}{l|l} & P \\ & \vdots \\ \triangleright & P \end{array}$$

Modus Tollens
(MT)

$$\begin{array}{l|l} & P \rightarrow Q \\ & \vdots \\ & \neg Q \\ & \vdots \\ \triangleright & \neg P \end{array}$$

Identity Introduction
($=$ Intro)

$$\begin{array}{l|l} \triangleright & n = n \end{array}$$

Identity Elimination
($=$ Elim)

$$\begin{array}{l|l} & P(n) \\ & \vdots \\ & n = m \\ & \vdots \\ \triangleright & P(m) \end{array}$$

General Conditional Proof
($\forall$ Intro)

$$\begin{array}{l|l} & \quad | \; \boxed{c} \; P(c) \\ & \quad | \; \vdots \\ & \quad | \; Q(c) \\ \triangleright & \forall x\,(P(x) \rightarrow Q(x)) \end{array}$$

Universal Elimination
($\forall$ Elim)

$$\begin{array}{l|l} & \forall x\, S(x) \\ & \vdots \\ \triangleright & S(c) \end{array}$$

Universal Introduction
($\forall$ Intro)

$$\begin{array}{l|l} & \quad | \; \boxed{c} \\ & \quad | \; \vdots \\ & \quad | \; P(c) \\ \triangleright & \forall x\, P(x) \end{array}$$

where c does not occur outside the subproof where it is introduced.

De Morgan Rules
(DeM)

$$\neg(P \wedge Q) \quad \triangleleft\triangleright \quad \neg P \vee \neg Q$$

$$\neg(P \vee Q) \quad \triangleleft\triangleright \quad \neg P \wedge \neg Q$$

Existential Introduction
($\exists$ Intro)

$$\begin{array}{l|l} & S(c) \\ & \vdots \\ \triangleright & \exists x\, S(x) \end{array}$$

Existential Elimination
($\exists$ Elim)

$$\begin{array}{l|l} & \exists x\, S(x) \\ & \vdots \\ & \quad | \; \boxed{c} \; S(c) \\ & \quad | \; \vdots \\ & \quad | \; Q \\ \triangleright & Q \end{array}$$

where c does not occur outside the subproof where it is introduced.