Philosophy 1102: Introduction to Logic

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2. Arguments

We saw in Section 1 that logic is the study of propositions, i.e. both thoughts and possible states of affairs. In this course we are mostly concerned with human thinking, or reasoning, and with *arguments* in particular.

2.1 What is an argument?

The purpose of an argument is to add more truth to one's stock of beliefs, hopefully without admitting any more false beliefs. Arguments, handled carefully, are a way of getting more knowledge.

The most important part of an argument is the *conclusion*. This is a proposition. The aim of an argument is to persuade the listener to believe the conclusion, i.e. to think that the conclusion is true. Not just any means of persuasion counts as an argument, however. Bullying, for instance, is a form of persuasion that is quite different from argument. Also, emotional appeals can be persuasive, but these are not arguments either. An argument is an attempt at *rational* persuasion, an appeal to the listener's logical faculty, helping him to see for himself that the conclusion must be true, or is at least likely to be true.

Usually an argument begins with some agreement between the two parties involved. They share some assumptions, or beliefs, in other words. The person making an argument will often appeal to some of these shared beliefs as evidence to support his case. These shared beliefs, which are accepted without argument, are known as *premises* of the argument. They are the starting point of the argument.

If two people have such different opinions on a particular subject that they hardly share any beliefs concerning it, then it will be difficult (if not impossible) for them to argue about that subject. Nearly every argument has premises, or starting assumptions, which must be accepted by all parties if the argument is to be persuasive.

Examples

1. The clearest, most rational arguments are those found in mathematics. Indeed, many people have learned to reason logically from a mathematical training. Here is an example of a mathematical argument that is easy for all to understand.

The argument concerns *prime* numbers. A prime number is a (positive) whole number that can only be divided by one and itself. For example, five is prime, since it can only be divided into one and five. Five will not divide (without remainder) into 2, 3 or 4. Six is not prime, as it can be divided into 2 or 3. The prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ...

One question is: Do the prime numbers ever end? Or do they go on forever, like the even numbers? We shall argue that the prime numbers go on forever.

Before we start, we need two simpler results that will be used to prove the main result. Such a supporting result is often called a *lemma*. For the first lemma we need the concept of a *factor*. A factor is a number that goes into another number exactly. So 4 is a factor of 12, 8 is a factor of 40, and so on. A prime number is then a number, 2 or greater, whose only factors are one, and itself.

Lemma 1 If a number has a factor (other than one or itself), then it has a prime factor.

This is obvious, when you realise that a factor of a factor is itself a factor. For example, 12 is a factor of 36, and 6 is a factor of 12, so 6 is a factor of 36. We can keep breaking each factor into smaller and smaller factors, but this can't go on forever. Eventually we must get down to a factor which has no factors itself. In other words, we will eventually find a prime factor.

Lemma 2 Let $a_1, a_2, a_3, ..., a_n$ be whole numbers that are greater than 1. Then $(a_1 \times a_2 \times a_3 \times ... \times a_n) + 1$ is not exactly divisible by any one of $a_1, a_2, a_3, ..., a_n$.

This is clearly true, as $(a_1 \times a_2 \times a_3 \times ... \times a_n)$ divides exactly into each of $a_1, a_2, a_3, ..., a_n$, and the "left over" unit cannot be divided by anything bigger than one.

Main Argument:

We are trying to show that the prime numbers go on forever, but (for the sake of argument) let us assume just the opposite. Let us assume that the prime numbers stop at some point. It follows that there is a highest prime. Call it p. Now consider the whole number N, which is the product of *all* the prime numbers, plus one, i.e.

$$N = (2 \times 3 \times 5 \times 7 \times ... \times p) + 1.$$

This number N is obviously much greater than p, so N cannot be prime. (As p is the highest prime.) So, since N has a factor, it must have a prime factor, according to Lemma 1. In other words, one of the numbers 2, 3, 5, ..., p is a factor of N. But this is ruled out by Lemma 2! So N does not have any prime factors, after all. It follows that N is a prime number. So p is not the highest prime number. We could use basically the same argument to show that N isn't the highest prime either. It becomes clear that, for any prime number, we can find a larger one. So there is no highest prime number.■

2.2 Invalid Arguments

I say that an argument is an *attempt* to convince someone using logic, because some arguments are said to be "invalid". This means that the argument is unsuccessful, in the sense that the conclusion does not logically follow from the premises. (It may still succeed in persuading the listener, however.) An invalid argument does not *properly* persuade the listener to believe the conclusion, as the grounds given are not adequate to justify any such belief. Anyone persuaded by an invalid argument is making a mistake. Though the conclusion may be true, they are mistaken in thinking that they have good grounds to believe it.

Consider the following mathematical argument.

> Assume, for the sake of argument, that $a = b$. It follows that $a^2 = ab$. Subtracting b^2 from both sides, we infer that $a^2 - b^2 = ab - b^2$. Factorising each side of the equation, we find that:
>
> $(a + b)(a - b) = b(a - b)$.
>
> Now, since the same factor $(a - b)$ is on both sides, we may cancel through to get:
>
> $a + b = b$.
>
> Since $a=b$, we can infer that $b + b = b$.
>
> This follows from the assumption that $a = b$, so it must be valid for all values of b. In particular, it must be valid for $b = 1$. Thus $1+1 = 1$, i.e. $2 = 1$.■

The conclusion of this argument, that 2 = 1, is clearly false. So the conclusion does not follow from the premise. Such an argument is called *invalid*. Invalid arguments are still arguments, even though they contain mistakes, because they do at least *attempt* to reason logically. By the way, what is the mistake in the argument above?

Here, in summary, is our definition of an argument:

> An argument is an attempt to persuade the listener that some proposition (the conclusion) is true. The arguer begins with some claims that the listener already accepts (the premises), and tries to show that, given these, the conclusion rationally follows and should be accepted.

2.3 Unsound Arguments

An invalid argument is a failure, as it does not (properly) succeed in its objective to persuade the listener that the conclusion is true. While the listener may be convinced by an invalid argument, this is a hollow achievement, since the listener's new belief is based on a mistake, and therefore not "above board". It is not knowledge.

Another way for an argument to fail is if the listener doesn't already accept the premises of the argument. In this case, the listener will not even bother to consider the rest of the argument, as it will seem pointless and irrelevant.

Suppose, for example, someone claims that the world is only about 6,000 years old. "Really?" you say. "Why should we think that?" So he offers an argument. "We know that Genesis chapters 1-11 are true, and must be interpreted quite literally", he says. "Moreover, the genealogies are complete, with no gaps at all." Then he continues with the argument. But most of us aren't listening any more, as we don't accept the premises. Some might not recognise the book of Genesis as true in any sense. Others may accept it as true, but not interpret it literally, and so on.

Note that, in order to find an argument compelling, you must accept *all* of the premises as true. The presence of even one dubious premise, if it's needed for the conclusion to follow, will make the argument suspect.

To say that an argument may be defective in some way, even if it's valid, we use the term *unsound*. There are two kinds of unsoundness, which may be called *objective* and *subjective*. An argument is objectively unsound if one or more of the premises is false. An argument is subjectively unsound if one or more of the premises is not believed, or accepted as true, by the listener.

Definition An *objectively sound* argument is one that is valid, and whose premises are all (objectively) true.

Definition A *subjectively sound* argument is one that is valid, and whose premises are all believed (accepted as true) by the listener.

Note that, in the subjective sense, an argument may be sound for one person, but unsound for another.

There are thus two ways for an argument to be unsound. First, it may be invalid. Second, the premises may be false (or not believed). Of course, an argument that has both defects is unsound as well.

The concept of subjective soundness is the more useful one, since when we construct arguments we always aim at subjective soundness. There is no point in using premises that the listener rejects, even if we think they're true. Moreover, we may even use premises that we ourselves reject, if the listener accepts them. This may seem insincere, but I think it's ok as long as you make it clear that this is what you're doing. Do you agree?

Note that, in most logic texts, the term "sound" means *objectively* sound.

Exercise 2.1

For each passage below, say whether it contains an argument. If it does, then say whether or not the argument is subjectively sound (for you). If it's unsound, then say whether the argument is invalid, or the premises unacceptable (or both). You don't have to justify your answer.

1. I never enjoyed playing sports at school. I don't like watching pro sports on TV either. In fact, I don't like sports, period.

2. There is a God. I believe this because that's how I was raised. I went to Sunday school every week, and read the Bible all the time.

3. At the present rate of consumption, the oil will be used up in 20 to 25 years. And we're sure not going to reduce consumption in the near future. So we'd better start developing solar power, windmills, and other "alternative" energy sources pretty soon.

4. The abortion issue is blown out of all proportion. How come we don't hear nearly as much about the evils of the Pill? After all, a lot more potential people are "killed" by the Pill than by abortion.

5. If God were all good he would want his creatures always to be happy. If God were all powerful, he would be able to accomplish anything he wants. Therefore, God must be lacking either in power or goodness or both.

2.4 Arguments and "standard form", or "Fitch Format"

In many logic textbooks, an argument is defined as something like the following:

> An argument is a series of statements, one of which is the conclusion, and the others are the premises.

In other words, they say that an argument looks like this:

1. First premis
2. Second premis
3. [more premises?]

∴ 4. Conclusion

Does this definition fit the arguments we've looked at? They do have premises, and a conclusion, but is that *all*? Suppose we consider the argument that 2=1. What are the premises of the argument? There is really no premise at all. What is the conclusion? The conclusion is that 2=1. So here is the argument, according to these texts:

[no premise]
$\therefore$ 1. $2 = 1$

Is this the argument we gave before? There seems to be quite a lot missing! In fact, all the logical reasoning is missing, which is rather an important part. Without that part we can have a huge jump, with no justification of the claim that the conclusion follows from the premises.

The argument contains a sub-argument, which infers that $b + b = 1$, from the assumption that $a = b$. So this sub-argument, in standard form, is:

1. $a = b$
$\therefore$ 2. $b + b = b$

Again, most of the argument is missing from this presentation.

In other philosophy classes, where you have to write essays, you are very often asked to present someone else's argument. When you do this, do *NOT* just state the premises and the conclusion! The "bit in between", the persuasive reasoning, is also needed.

Logic textbooks use the term 'argument' in a special, technical, sense used only by logicians. Sometimes we use the term 'standard form' to describe this way of presenting an argument. If you only give the premises and conclusion of the argument, we say that you are *presenting the argument in standard form, or in Fitch Format*.

Exercise 2.2

1. For each argument in the previous exercise, write the argument in standard form

2. Write Betty's argument, from the dialogue below, in standard form.

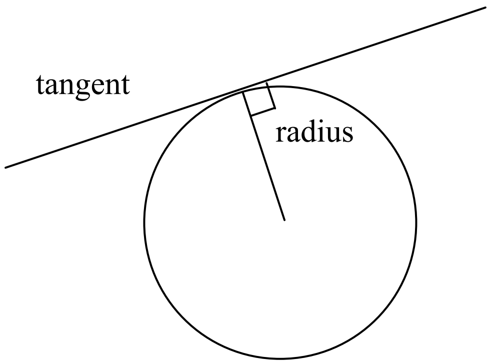

Figure 1

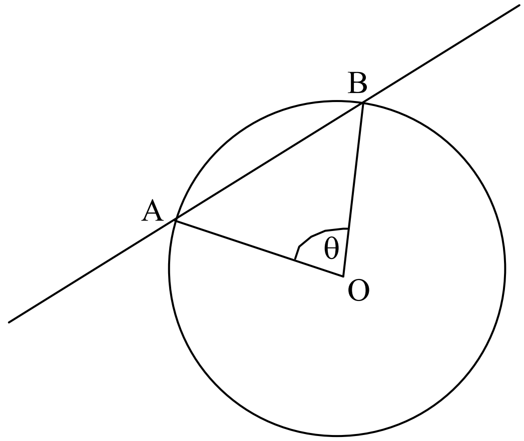

Figure 2

Betty: A tangent to a circle is always at right angles to the radius of the circle, as in Figure 1.

Fred: How do you know that?

Betty: Think about an extended chord AB, as in Figure 2. Since OA and OB are both radii, they're the same length. This means that OAB is an isosceles triangle.

Fred: Oh, right. An isosceles triangle is one with two sides that are the same length.

Betty: Now we know that the base angles of an isosceles triangle are equal, so that $\angle OAB = \angle OBA$.

Fred: Of course. I remember the professor saying that, and it makes sense.

Betty: Now suppose, just for the sake of argument, that $\angle AOB$ is θ degrees. Since we also know that the angles inside a triangle always add up to 180 degrees, we can see that $\angle OAB = \angle OBA = (90 - ½\theta)$ degrees.

Fred: Fair enough. But where are you going with this?

Betty: The trick now is to let θ tend to zero, so that A and B get closer together. In the limit, when A and B merge, θ will equal 0. The extended chord AB will then be a tangent, and the angle between the tangent and OA will be exactly 90 degrees, i.e. a right angle.

2.5 Validity

The term 'valid', in logic, is a kind of moral term, meaning roughly "good" or "correct". A valid argument is, roughly speaking, one where the premises give enough information to guarantee the conclusion. Note that the term valid is *only* applied to arguments, not to sentences. Sentences are true or false; arguments are valid or invalid.

A common textbook definition of validity is as follows:

> An argument is valid just in case it is impossible for all of its premises to be true and yet its conclusion be false.

This is good as far as it goes, but we need to understand what is meant by "impossible". In Section 1 we talked about belief and truth, and subjective and objective meaning. Well, there's also a difference between subjective and objective impossibility. Consider the following argument:

1. Hesperus has one moon
∴ 2. Phosphorus has one moon

Suppose Ralph, our ancient astronomer, were to make this argument. Would it be correct? No it wouldn't, as Ralph has no evidence that Hesperus is Phosphorus. He has no right to draw that conclusion, from the premise stated. But is the argument valid? It looks like it might be, according to the textbook definition of validity. For, since Hesperus is *in fact* Phosphorus, it's objectively impossible for the premise to be true and the conclusion false.

All logicians would want to say that Ralph's argument is invalid, so we need some other understanding of the term "impossible". Can you think of something suitable? I think we should understand it as *subjective* impossibility. The reason the argument is invalid is that, *for all Ralph knows*, the premise could be true and the conclusion false. It's true that, in reality, Hesperus is Phosphorus. But Ralph doesn't know this, so it's not relevant. When he makes inferences, he has to work with what he knows.

For another example like this, consider a person who knows what hydrogen is, but doesn't know that water is H_2O, or even that water is made of hydrogen and oxygen. Then suppose they make the following argument.

1. This glass contains water.
∴ 2. This glass contains hydrogen.

Is the argument valid? No, because *for all that person knows*, it's possible for the premise to be true and the conclusion false. Note that, objectively speaking, it's impossible for the premise to be true and the conclusion false.

2.6 Valid Argument vs. Cogent Reasoning

There's another tricky point about validity that you need to understand. Whether or not an argument is valid depends *only* on what the premises and conclusion are. The "bit in between", the persuasive reasoning, has nothing to do with it. In other words, to determine whether or not an argument is valid you only need to see it in standard form.

Example

Is this argument valid?

1. The name ‘Arkansas’ for Arkansas has been used for decades.
2. Names that have been used for decades should not be changed.

> You see, if we change the name of Arkansas, then it won’t be Arkansas no more – so I won’t be able to live in Arkansas no more. But I’ve been living here all my life, and don’t wanna go live someplace else. Names should never be changed, under any circumstances. Mind you, if the name were changed, then I’d do whatever I could to change it back.

$\therefore$ 3. The name ‘Arkansas’ for Arkansas should not be changed.

You might think that it the argument is invalid, as the reasoning used is quite ridiculous. It is quite illogical. However, this argument is valid. If you ignore the reasoning, and just look at the premises and conclusion, then you can see that it’s valid. The two premises, put together, do guarantee the conclusion. A rational person who accepted the premises would also accept the conclusion.

If the argument above counts as valid, then what word can we use to describe wrong with it? We can say that the reasoning isn’t *cogent*, or *persuasive*.

Here’s another example. In 1995 Andrew Wiles published a proof of Fermat’s last theorem, which entails that there are no whole numbers a, b and c such that $a^3 + b^3 = c^3$. Assuming that the theorem is true, the following argument is valid:

$a^3 + b^3 = c^3$

Hence, a, b and c are not all whole numbers

But is the argument cogent, or persuasive? Not at all, for it doesn’t give the slightest clue about how we might infer the conclusion from the premise. (Wiles’s proof was over 150 pages long!)

2.6 Arguments and Explanations

It’s easy to mix up arguments with explanations, as they can look quite similar. To see the difference here, we should first distinguish between two different relations:

(i) The entailment (or logical consequence) relation

(ii) The cause-effect relation.

The entailment relation is a logical relation, and so is a relation between propositions. One proposition *A* entails another one *B* just in case the truth of *A* guarantees the truth of *B*. In other words, *A* provides conclusive evidence for *B*. If you know that *A* is true, then you can *infer* (come to believe) *B* with certainty. If *A* is true, then it follows that *B* is true. For example, "Ralph is six feet tall" entails "Ralph is more than 5 foot 6 tall".

The cause-effect relation is physical rather than logical. We say that one physical event *C* causes another one, *E*, just in case *C* brings *E* about, or makes it happen. *C* is called the cause, and *E* is the effect. For example, Ralph's taking growth hormones may be the cause of his being six feet tall. A fire may have been caused by faulty wiring that became hot. A child is caused by its parents, and so on. An explanation can always be thought of as an answer to a "Why?" question. Why did the house catch fire? Because of faulty wiring. Why does this child exist? Because this woman and man produced her.

(Note that an argument can be an answer to a "Why should I *believe* this?" question.)

The word 'because' is rather tricky, as it sometimes describes a cause-effect relation, and sometimes a relation of entailment. Consider these two claims:

(i) Charlie is sick because he ate lobster yesterday.
(ii) Charlie is sick, because he is still in bed.

Which of these describes a cause-effect relation, and which one a logical consequence relation?

The first one seems to take it for granted that Charlie is sick. It is saying that the illness was caused by his eating lobster the day before. Perhaps Charlie is allergic to seafood, or maybe the lobster contained salmonella bacteria.

The second one is trying to persuade us that Charlie is sick. The premise is that Charlie is still in bed. The idea no doubt is that Charlie always gets up early, except when he is ill. So, since it is late and he's not up yet, it follows that he's sick.

Note that, in (ii), there is no claim that his being in bed *caused* him to become sick. People don't usually get sick from staying in bed for an extra couple of hours. Rather, it's the evidence that allows us to *infer* that he's sick. In (i), on the other hand, it's assumed we already know that he is sick. There is no attempt to persuade us of that. The idea that Charlie ate lobster is not supposed to make us believe that he's sick, but is claimed to be the *cause* of his sickness.

Which one of (i) and (ii) contains an argument, and which is an explanation? An argument is an attempt to persuade using reasons. It claims that the premises, that have been given, logically entail the conclusion. An explanation is a story about what caused an event that is agreed to have occurred.

Exercise 2.3

1. In each of the following sentences, say whether the word 'because' expresses a relation of logical consequence, or of cause and effect. State the premise and conclusion, or cause and effect, as appropriate.

a. God exists, because otherwise life would be meaningless.
b. The bank collapsed because of the heavy rain last week.
c. Abortion is not wrong, because a woman should be able to control her own body.
d. The moon *was* full last night because I saw it!
e. Canada's softwood lumber industry is suffering as a result of the US import tariff.

2. For each of the following passages, say if it expresses an argument or an explanation. If it is an argument, then write it in standard form. If it is an explanation, state the cause and effect (or write down the causal sequence of events).

a. Why is a sodium flame yellow? Because of the ionisation energy of sodium atoms. Photons of this energy have a wavelength that we see as yellow.

b. Why should you believe that the earth is warming up? Because the concentration of CO_2 is up, and in the past this has always meant higher temperatures.

c. The dinosaurs died out as a result of global climate change. This was most likely due a large meteor impact, that would have put a lot of dust in the upper atmosphere, blocking the sun.

d. The dinosaurs died out as a result of global climate change. The fossil record shows that it happened very suddenly, and there were no other animals around that could have forced them into extinction. Mammals didn't really spread until the dinosaurs were already gone. It had to be severe climate change, as nothing else could have done it.

3. Consider the following two conversations.

Fred: I don't have to worry about getting the flu this year.

Betty: It's nice to see such optimism. But the sad truth is that the flu can strike anyone down.

Fred: No, really. I just had the flu shot. I'm safe.

Fred: I don't have to worry about getting the flu this year.

Betty: That's great! How did you manage that?

Fred: It was simple: I just got the flu shot.

In one conversation Fred offers an argument, and in the other he provides an explanation. Which is which?