Philosophy 1102: Introduction to Logic

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Logical and Formal Relations (and Properties)

Here are the main properties and relations that logicians are interested in:

Logical Relation/Property	**Symbols**	**Meaning**
Q is a logical consequence of P (P logically entails Q)	$P \Rightarrow Q$	There's no possible world in which P is true and Q is false.
P is logically necessary (P is logically true, or analytic)	$\Box P$ $\{\} \Rightarrow P$	P is true in all possible worlds
P is logically possible	$\Diamond P$ $\neg\Box\neg P$	P is true in at least one possible world
P is logically contingent	$\Diamond P \wedge \neg\Box P$, $\Diamond P \wedge \Diamond\neg P$	P is true in at least one, but not all, possible worlds
P is logically equivalent to Q	$P \Leftrightarrow Q$	P has the same truth value as Q in all possible worlds
P is logically inconsistent with Q	$P \Rightarrow \neg Q$, $\neg\Diamond(P \wedge Q)$	There's no possible world in which P and Q are both true

Note that these "symbols", while used by logicians, are not part of the language FOL.

The table of formal "TT" relations is almost identical; the only difference is that "possible world" is replaced with "row of the truth table".

Formal Relation/Property	**Symbols**	**Meaning**
Q is a TT consequence of P	$P \vDash Q$ $P \Rightarrow_{TT} Q$	There's no row of the truth table in which P is true and Q is false.
P is TT necessary (a tautology)	$\Box_{TT}P$ $\{\} \vDash P$	P is true in all rows of the truth table
P is TT possible	$\Diamond_{TT}P$	P is true in at least one row of the truth table
P is TT equivalent to Q	$P \Leftrightarrow_{TT} Q$	P has the same truth value as Q in all rows of the truth table
P is TT inconsistent with Q	$P \vDash \neg Q$, $\neg\Diamond_{TT}(P \wedge Q)$	There's no row of the truth table in which P and Q are both true

Note that these "symbols", while used by logicians, are not part of the language FOL.

If we compare the properties *logically necessary* and *tautology* (or TT necessary) then they look very similar. The difference between them arises from the fact that some rows of the truth table may describe impossible worlds. Hence P may be false in some row of its truth table, even if P is true in all possible worlds.

An example of such a sentence is ¬Older(fred, fred). Fred cannot possibly be older than himself, so this sentence is true in all possible worlds. But, through the Boolean Goggles, the sentence (structure) is ¬A. The truth table for this sentence is:

A	¬A
T	F
F	T

We see that the sentence ¬A is false in the first row, so ¬A, which is really ¬Older(fred, fred), is not a tautology. On the other hand, any sentence which *is* a tautology is automatically a logical necessity is well.

Examples of TT necessary, logically necessary, sentences.

	logically necessary?	**TT necessary?**	**Remarks**
Tet(a) ∧ Cube(b)	✘	✘	logically contingent
Tet(a) ∧ Cube(a)	✘	✘	logically impossible
1+1 = 2	✓	✘	
¬(Cube(a) ∧ ¬Cube(a))	✓	✓	
¬Larger(a, b) ∨ Smaller(b, a)	✓	✘	

Hints:

(i) P is necessary ⇔ ¬P is impossible

(and P is contingent ⇔ ¬P is contingent)

(works for both logical and TT necessity)

So if the sentence is a negation, i.e. of the form ¬P, then just look at P. If P is impossible, the ¬P is necessary. If P is contingent, then ¬P is contingent as well, so that ¬P isn't necessary.

(ii) Sometimes you can use a *de Morgan* rule to convert the sentence to a negation, i.e. "pull out the ¬". Then use hint (i).