

Rules of Inference in \mathcal{F}

Conjunction Introduction
(\wedge Intro)

$$\begin{array}{|l} P_1 \\ \downarrow \\ P_n \\ \vdots \\ \triangleright P_1 \wedge \dots \wedge P_n \end{array}$$

Conjunction Elimination
(\wedge Elim)

$$\begin{array}{|l} P_1 \wedge \dots \wedge P_i \wedge \dots \wedge P_n \\ \vdots \\ \triangleright P_i \end{array}$$

Negation Introduction
(\neg Intro)

$$\begin{array}{|l} P \\ \vdots \\ \perp \\ \triangleright \neg P \end{array}$$

Negation Elimination
(\neg Elim)

$$\begin{array}{|l} \neg \neg P \\ \vdots \\ \triangleright P \end{array}$$

Disjunction Introduction
(\vee Intro)

$$\begin{array}{|l} P_i \\ \vdots \\ \triangleright P_1 \vee \dots \vee P_i \vee \dots \vee P_n \end{array}$$

Disjunction Elimination
(\vee Elim)

$$\begin{array}{|l} P_1 \vee \dots \vee P_n \\ \vdots \\ P_1 \\ \vdots \\ S \\ \downarrow \\ P_n \\ \vdots \\ S \\ \triangleright S \end{array}$$

\perp Introduction
(\perp Intro)

$$\begin{array}{|l} P \\ \vdots \\ \neg P \\ \vdots \\ \triangleright \perp \end{array}$$

\perp Elimination
(\perp Elim)

$$\begin{array}{|l} \perp \\ \vdots \\ \triangleright P \end{array}$$

Reiteration
(Reit)

$$\begin{array}{|l} P \\ \vdots \\ \triangleright P \end{array}$$

Conditional Introduction
(\rightarrow Intro)

$$\begin{array}{|l} P \\ \vdots \\ Q \\ \triangleright P \rightarrow Q \end{array}$$

Conditional Elimination
(\rightarrow Elim)

$$\begin{array}{|l} P \rightarrow Q \\ \vdots \\ P \\ \vdots \\ \triangleright Q \end{array}$$

Biconditional Introduction
(\leftrightarrow Intro)

$$\begin{array}{|l} P \\ \vdots \\ Q \\ \vdots \\ Q \\ \vdots \\ P \\ \triangleright P \leftrightarrow Q \end{array}$$

Biconditional Elimination
(\leftrightarrow Elim)

$$\begin{array}{|l} P \leftrightarrow Q \text{ (or } Q \leftrightarrow P) \\ \vdots \\ P \\ \vdots \\ \triangleright Q \end{array}$$

Identity Introduction
(= Intro)

$$\triangleright n = n$$

Identity Elimination
(= Elim)

$$\begin{array}{|l} P(n) \\ \vdots \\ n = m \\ \vdots \\ \triangleright P(m) \end{array}$$