planet has returned to $P$ again. This time the planet is seen silhouetted at 3 , to the east of position 2. It has completed more than one journey around the ecliptic while moving only once through its orbit, and its second journey around the ecliptic was therefore a very rapid one. After a third revolution the planet is again at $P$, but it appears at position 4, east of 3 , and its journey around the ecliptic was therefore


Figure 33. The Copernican explanation of variations in the time required for a superior planet to complete successive journeys around the ecliptic. While the planet moves once eastward around its orbit from $P$ to $P$, the earth makes $11 / 4$ eastward revolutions from $E_{1}$ to $E_{1}$ and on to $E_{2}$. During this interval the apparent position of the planet among the stars moves eastward from 1 to 2 , slightly less than a full trip. During the planet's next revolution the earth moves from $E_{2}$ to $E_{2}$ and on to $E_{3}$, so that its apparent position among the stars shifts from 2 to 1 and on to 1 again, slightly more than one full trip around the ecliptic.
again a fast one. After a fourth revolution in its orbit the planet again appears at 1 , west of 4 , and its final trip was therefore slow. The planet has completed four trips about its orbit and four trips around the ecliptic at the same instant. The average time required by a superior planet to circle the ecliptic is therefore identical with the planet's orbital period. But the time required for an individual trip may be considerably greater or considerably less than the average. A similar argument will account for the similar irregularities of an inferior planet's motion.

Retrograde motion and the variation of the time required to circle
the ecliptic are the two gross planetary irregularities which in antiquity had led astronomers to employ epicycles and deferents in treating the problem of the planets. Copernicus' system explains these same gross irregularities, and it does so without resorting to epicycles, or at least to major epicycles. To gain even an approximate and qualitative account of the planetary motions Hipparchus and Ptolemy had required twelve circles - one each for the sun and moon, and two each for the five remaining "wanderers." Copernicus achieved the same qualitative account of the apparent planetary motions with only seven circles. He needed only one sun-centered circle for each of the six known planets - Mercury, Venus, Earth, Mars, Jupiter, and Saturn - and one additional earth-centered circle for the moon. To an astronomer concerned only with a qualitative account of the planetary motions, Copernicus' system must seem the more economical.

But this apparent economy of the Copernican system, though it is a propaganda victory that the proponents of the new astronomy rarely failed to emphasize, is largely an illusion. We have not yet begun to deal with the full complexity of Copernicus' planetary astronomy. The seven-circle system presented in the First Book of the De Revolutionibus, and in many modern elementary accounts of the Copernican system, is a wonderfully economical system, but it does not work. It will not predict the position of planets with an accuracy comparable to that supplied by Ptolemy's system. Its accuracy is comparable to that of a simplified twelve-circle version of Ptolemy's system - Copernicus can give a more economical qualitative account of the planetary motions than Ptolemy. But to gain a reasonably good quantitative account of the alteration of planetary position Ptolemy had been compelled to complicate the fundamental twelve-circle system with minor epicycles, eccentrics, and equants, and to get comparable results from his basic seven-circle system Copernicus, too, was forced to use minor epicycles and eccentrics. His full system was little if any less cumbersome than Ptolemy's had been. Both employed over thirty circles; there was little to choose between them in economy. Nor could the two systems be distinguished by their accuracy. When Copernicus had finished adding circles, his cumbersome sun-centered system gave results aśs accurate as Ptolemy's, but it did not give more accurate results. Copernicus did not solve the problem of the planets.

The full Copernican system is described in the latter books of the De Revolutionibus. Fortunately we need only illustrate the sorts of complexities there developed. Copernicus' system was not, for example, really a sun-centered system at all. To account for the increased rate at which the sun travels through the signs of the zodiac during the winter, Copernicus made the earth's circular orbit eccentric, displacing its center from the sun's. To account for other irregularities, indicated by ancient and contemporary observations of the sun's motion, he kept this displaced center in motion. The center of the earth's eccentric was placed upon a second circle whose motion continually varied the extent and direction of the earth's eccentricity. The final system employed to compute the earth's motion is represented approximately in Figure $34 a$. In the diagram, $S$ is the sun, fixed in space; the point $O$, which itself moves slowly about the sun, is the center of a slowly rotating circle that carries the moving center $O_{B}$ of the earth's eccentric; $E$ is the earth itself.

Similar complexities were necessitated by the observed motions of the other heavenly bodies. For the moon Copernicus used a total of three circles, the first centered on the moving earth, the second centered on the moving circumference of the first, and the third on the


Figure 34. Copernicus' account of the motion of (a) the earth and (b) Mars. In (a) the sun is at $S$, and the earth, $E$, revolves on a circle whose center, $O_{s,}$, revolves slowly about a point $O$, which in turn revolves on a sun-centered circle. In (b) Mars is placed on an epicycle revolving on a deferent whose center, $O_{x}$, maintains a fixed geometric relation to the moving center $O_{B}$ of the earth's orbit.
circumference of the second. For Mars and most of the other planets he employed a system much like that illustrated in Figure 34b. The center of Mars's orbit, $O_{M}$, is displaced from the center of the earth's orbit, $O_{B}$, and is moved with it; the planet itself is placed at $M$, not on the eccentric but on an epicycle, which rotates eastward in the same direction and with the same period as the eccentric. Nor do the complexities end here. Still other devices, fully equivalent to Ptolemy's, were required to account for the north and south deviations of each planet from the ecliptic.

Even this brief sketch of the complex system of interlocking circles employed by Copernicus to compute planetary position indicates the third great incongruity of the De Revolutionibus and the immense irony of Copernicus' lifework. The preface to the De Revolutionibus opens with a forceful indictment of Ptolemaic astronomy for its inaccuracy, complexity, and inconsistency, yet before Copernicus' text closes, it has convicted itself of exactly the same shortcomings. Copernicus' system is neither simpler nor more accurate than Ptolemy's. And the methods that Copernicus employed in constructing it seem just as little likely as the methods of Ptolemy to produce a single consistent solution of the problem of the planets. The De Revolutionibus itself is not consistent with the single surviving early version of the system, described by Copernicus in the early manuscript Commentariolus. Even Copernicus could not derive from his hypothesis a single and unique combination of interlocking circles, and his successors did not do so. Those features of the ancient tradition which had led Copernicus to attempt a radical innovation were not eliminated by that innovation. Copernicus had rejected the Ptolemaic tradition because of his discovery that "the Mathematicians are inconsistent in these [astronomical] investigations" and because "if their hypotheses were not misleading, all inferences based thereon might surely be verified." A new Copernicus could have turned the identical arguments against him.

## The Harmony of the Copernican System

Judged on purely practical grounds, Copernicus' new planetary system was a failure; it was neither more accurate nor significantly simpler than its Ptolemaic predecessors. But historically the new sys ${ }_{\text {w }}$
tem was a great success; the De Revolutionibus did convince a few of Copernicus' successors that sun-centered astronomy held the key to the problem of the planets, and these men finally provided the simple and accurate solution that Copernicus had sought. We shall examine their work in the next chapter, but first we must try to discover why they became Copernicans - in the absence of increased economy or precision, what reasons were there for transposing the earth and the sun? The answer to this question is not easily disentangled from the technical details that fill the De Revolutionibus, because, as Copernicus himself recognized, the real appeal of suncentered astronomy was aesthetic rather than pragmatic. To astronomers the initial choice between Copernicus' system and Ptolemy's could only be a matter of taste, and matters of taste are the most difficult of all to define or debate. Yet, as the Copernican Revolution itself indicates, matters of taste are not negligible. The ear equipped to discern geometric harmony could detect a new neatness and coherence in the sun-centered astronomy of Copernicus, and if that neatness and coherence had not been recognized, there might have been no Revolution.

We have already examined one of the aesthetic advantages of Copernicus' system. It explains the principal qualitative features of the planetary motions without using epicycles. Retrograde motion, in particular, is transformed to a natural and immediate consequence of the geometry of sun-centered orbits. But only astronomers who valued qualitative neatness far more than quantitative accuracy (and there were a few - Galileo among them) could consider this a convincing argument in the face of the complex system of epicycles and eccentrics elaborated in the De Revolutionibus. Fortunately there were other, less ephemeral, arguments for the new system. For example, it gives a simpler and far more natural account than Ptolemy's of the motions of the inferior planets. Mercury and Venus never get very far from the sun, and Ptolemaic astronomy accounts for this observation by tying the deferents of Mercury, Venus, and the sun together so that the center of the epicycle of each inferior planet always lies on a straight line between the earth and the sun (Figure $35 a$ ). This alignment of the centers of the epicycles is an "extra" device, an ad $h o c$ addition to the geometry of earth-centered astronomy, and there is no need for such an assumption in Copernicus' system. When, as in

Figure $35 b$, the orbit of a planet lies entirely within the earth's orbit, there is no way in which the planet can appear far from the sun. Maximum elongation will occur when, as in the diagram, the line from the earth to the planet is tangent to the planet's orbit and the angle SPE is a right angle. Therefore the angle of elongation, SEP, is the largest angle by which the inferior planet can deviate from the sun. The basic geometry of the system fully accounts for the way in which Mercury and Venus are bound to the sun.


Figure 35. Limited elongation of inferior planets explained in (a) the Ptolemaic and ( $b$ ) the Copernican systems. In the Ptolemaic system the angle between the sun, $S$, and the planet, $P$, must be restricted by keeping the center of the epicycle on the line between the earth and the sun. In the Copernican system, with the planet's orbit entirely contained by the earth's, no such restriction is necessary.

Copernican geometry illuminates another even more important aspect of the behavior of the inferior planets, namely, the order of their orbits. In the Ptolemaic system the planets were arranged in earth-centered orbits so that the average distance between a planet and the earth increased with the time required for the planet to traverse the ecliptic. The device worked well for the superior planets and for the moon, but Mercury, Venus, and the sun all require 1 year for an average journey around the ecliptic, and the order of their orbits had therefore always been a source of debate. In the Copernican system there is no place for similar debate; no two planets have the same orbital period. The moon is no longer involved in the problem, for it travels about the earth rather than about the central sun. The
superior planets, Mars, Jupiter, and Saturn, preserve their old order about the new center, because their orbital periods are the same as the average lengths of time they need to circle the ecliptic. The earth's orbit lies inside of Mars's, since the earth's orbital period, 1 year, is less than Mars's 687 days. It only remains to place Mercury and Venus in the system, and their order is, for the first time, uniquely determined.

This can be seen as follows. Venus is known to retrogress every 584 days, and since retrograde motion can be observed only when Venus passes the earth, 584 days must be the time Venus requires to lap the earth once in their common circuit of the sun. Now in 584 days the earth has traversed its orbit $\frac{584}{365}\left(=1 \frac{219}{365}\right)$ times. Since Venus has lapped the earth once during this interval, it must have circled its orbit $2 \frac{219}{365}\left(=\frac{949}{365}\right)$ times in just 584 days. But a planet that circles its orbit $\frac{949}{365}$ times in 584 days must require $584 \times \frac{365}{949}(=225)$ days to circle its orbit once. Therefore, since Venus's period, 225 days, is less than earth's, Venus's orbit must be inside the earth's, and there is no ambiguity. A similar calculation places Mercury's orbit inside Venus's and closest to the sun. Since Mercury retrogresses, and therefore laps the earth, every 116 days, it must complete its orbit just $1 \frac{116}{365}\left(=\frac{481}{365}\right)$ times in 116 days. Therefore it will complete its orbit just once in $116 \times \frac{365}{481}$ $(=88)$ days. Its orbital period of 88 days is the shortest of all, and it is therefore the planet closest to the sun.

So far we have ordered the sun-centered planetary orbits with the same device used by Ptolemaic astronomers to order earth-centered orbits: planets farther from the center of the universe take longer to circle the center. The assumption that the size of the orbit increases with orbital period can be applied more fully in the Copernican than in the Ptolemaic system, but in both systems it is initially arbitrary. It seems natural that planets should behave this way, like Vitruvius' ants on a wheel, but there is no necessity that they do so. Perhaps the assumption is entirely gratuitous, and the planets, excepting the sun and moon, whose distances can be directly determined, have another order.

The response to this suggested reordering constitutes another very important difference between the Copernican and the Ptolemaic systems, and one which, as we discovered in his preface, Copernicus
himself particularly emphasizes. In the Ptolemaic system the deferent and epicycle of any one planet can be shrunk or expanded at will without affecting either the sizes of the other planetary orbits or the position at which the planet, viewed from a central earth, appears against the stars. The order of the orbits may be determined by assuming a relation between size of orbit and orbital period. In addition, the relative dimensions of the orbits may be worked out with the aid of the further assumption, discussed in Chapter 3, that the minimum distance of one planet from the earth is just equal to the maximum distance between the earth and the next interior planet. But though both of these seem natural assumptions, neither is necessary. The Ptolemaic system could predict the same apparent positions for the planets without making use of either. In the Ptolemaic system the appearances are not dependent upon the order or the sizes of the planetary orbits.

There is no similar freedom in the Copernican system. If all the planets revolve in approximately circular orbits about the sun, then both the order and the relative sizes of the orbits can be determined directly from observation without additional assumptions. Any change in order or even in relative size of the orbits will upset the whole system. For example, Figure $36 a$ shows, an inferior planet, $P$, viewed from the earth at the time when it reaches its maximum elongation from the sun. The orbit is assumed circular, and the angle $\dot{S} P E$ must therefore be a right angle when the angle of elongation, $S E P$, reaches its maximum value. The planet, the sun, and the earth form a right triangle one of whose acute angles, $S E P$, can be directly measured. But knowledge of one acute angle of a right triangle determines the ratio of the lengths of the sides of that triangle. Therefore the ratio of the radius of the inferior planet's orbit, $S P$, to the radius of the earth's orbit, $S E$, can be computed from the measured value of the angle $S E P$. The relative sizes of the earth's orbit and the orbits of both inferior planets can be discovered from observation.

An equivalent determination can be made for a superior planet, though the techniques are more complex. One possible technique is illustrated in Figure 36b. Suppose that at some determined instant of time the sun, the earth, and the planet all lie on the straight line $S E P$; this is the orientation in which the planet lies diametrically across the ecliptic from the sun and is in the middle of a retrograde motion. Since the earth traverses its orbit more rapidly than any su-

(a)

(b)

Figure 36. Determining the relative dimensions of orbits in the Copernican system: (a) for an inferior planet; (b) for a superior planet.
perior planet, there must be some later instant of time when the earth at $E^{\prime}$ and the planet at $P^{\prime}$ will form a right angle $S E^{\prime} P^{\prime}$ with the sun, and since $S E^{\prime} P^{\prime}$ is the angle between the sun and the superior planet viewed from the earth, it can be directly determined and the time required to achieve it can be measured. The angle ESE' can now be determined, for it must bear the same ratio to $360^{\circ}$ as the time required by the earth to move from $E$ to $E^{\prime}$ bears to the 365 days that the earth requires to complete its orbit. The angle PSP' can be determined in just the same way, since the time required by the planet to complete its orbit is already known, and the time occupied by the planet in going from $P$ to $P^{\prime}$ is the same as that needed by the earth to go from $E$ to $E^{\prime}$. With $P S P^{\prime}$ and $E S E^{\prime}$ known, the angle $P^{\prime} S E^{\prime}$ can be found by subtraction. Then we again have a right triangle, $S E^{\prime} P^{\prime}$, with one acute angle, $P^{\prime} S E^{\prime}$, known, and the ratio of the radius of the planet's orbit, $S P^{\prime}$, to that of the earth's orbit, $S E^{\prime}$, can therefore be determined just as for an inferior planet.
By techniques like this the distances to all the planets can be determined in terms of the distance between the earth and the sun, or in terms of any unit, like the stade, in which the radius of the earth's orbit has been measured. Now, for the first time, as Copernicus says in his prefatory letter, "the orders and magnitudes of all stars and spheres . . . become so bound together that nothing in any part thereof could be moved from its place without producing confusion of all the other parts and of the universe as a whole." Because the
relative dimensions of the planetary orbits are a direct consequence of the first geometric premises of sun-centered astronomy, the new astronomy has for Copernicus a naturalness and coherence that were lacking in the older earth-centered version. The structure of the heavens can be derived from Copernicus' system with fewer extraneous or ad hoc assumptions like plenitude. That is the new and aesthetic harmony which Copernicus emphasizes and illustrates so fully in the tenth chapter of his introductory First Book, to which we now turn, having first learned enough about the new system (as Copernicus' lay readers had not) to understand what he is talking about.

## 10. Of the Order of the Heavenly Bodies.

No one doubts that the Sphere of the Fixed Stars is the most distant of visible things. As for the order of the planets, the early Philosophers wished to determine it from the magnitude of their revolutions. They adduce the fact that of objects moving with equal speed, those farther distant seem to move more slowly (as is proved in Euclid's Optics). They think that the Moon describes her path in the shortest time because, being nearest to the Earth, she revolves in the smallest circle. Farthest they place Saturn, who in the longest time describes the greatest circuit. Nearer than he is Jupiter, and then Mars.

Opinions differ as to Venus and Mercury which, unlike the others, do not altogether leave the Sun. Some place them beyond the Sun, as Plato in Timaeus; others nearer than the Sun, as Ptolemy and many of the moderns. Alpetragius [a twelfth-century Moslem astronomer] makes Venus nearer and Mercury farther than the Sun. If we agree with Plato in thinking that the planets are themselves dark bodies that do but reflect light from the Sun, it must follow, that if nearer than the Sun, on account of their proximity to him they would appear as half or partial circles; for they would generally reflect such light as they receive upwards, that is toward the Sun, as with the waxing or waning Moon. [See the discussion of the phases of Venus in the next chapter. Neither this effect nor the following is distinctly visible without the telescope.] Some think that since no eclipse even proportional to their size is ever caused by these planets, they can never be between us and the Sun. . . . [Copernicus proceeds to note many difficulties in the arguments usually used to determine the relative order of the sun and the inferior planets. Then he continues:]

Unconvincing too is Ptolemy's proof that the Sun moves between those bodies that do and those that do not recede from him completely [that is, between the superior planets which can assume any angle of elongation and the inferior planets whose maximum elongation is limited]. Con-
sideration of the case of the Moon, which does so recede, exposes its falseness. Again, what cause can be alleged, by those who place Venus nearer than the Sun, and Mercury next, or in some other order? Why should not these planets also follow separate paths, distinct from that of the Sun, as do the other planets [whose deferents are not tied to the sun's]? And this might be said even if their relative swiftness and slowness did not belie their alleged order. Either then the Earth cannot be the center to which the order of the planets and their Spheres is related, or certainly their relative order is not observed, nor does it appear why a higher position should be assigned to Saturn than to Jupiter, or any other planet.

Therefore I think we must seriously consider the ingenious view held by Martianus Capella [a Roman encyclopedist of the fifth century who recorded a theory of the inferior planets probably first suggested by Heraclides] . . . and certain other Latins, that Venus and Mercury do not go round the Earth like the other planets but run their courses with the Sun as center, and so do not depart from him farther than the convexity of their Spheres allows. . . . What else can they mean than that the center of these Spheres is near the Sun? So certainly the circle of Mercury must be within that of Venus, which, it is agreed, is more than twice as great.

We may now extend this hypothesis to bring Saturn, Jupiter and Mars also into relation with this center, making their Spheres great enough to contain those of Venus and Mercury and the Earth. . . . These outer planets are always nearer to the Earth about the time of their evening rising, that is, when they are in oppcsition to the Sun, and the Earth between them and the Sun. They are more distant from the Earth at the time of their evening setting, when they are in conjunction with the Sun and the Sun between them and the Earth. These indications prove that their center pertains rather to the Sun than to the Earth, and that this is the same center as that to which the revolutions of Venus and Me cury are related.
[Copernicus' remarks do not actually "prove" a thing. The Ptolemaic system explains these phenomena as completely as the Copernican, but the Copernican explanation is again more natural, for, like the Copernican explanation of the limited elongation of the inferior planets, it depends only on the geometry of a sun-centered astronomical system, not on the particular orbital periods assigned to the planets. Copernicus' remarks will be clarified by reference to Figure 32a. A superior planet retrogresses when the earth overtakes it, and under these circumstances it must be simultaneously closest to the earth and across the ecliptic from the sun. In the Ptolemaic system a retrogressing superior planet must be closer to the earth than at any other time, and it is in fact also across the sky from the sun. But it is only across the sky from the sun because the rates of rotation of its deferent and epicycle have particular values that happen to put the planet back in opposition to the sun whenever the epicycle brings
the planet back close to the central earth. If, in the Ptolemaic system, the period of epicycle or deferent were quantitatively slightly different, then the qualitative regularity that puts a retrogressing superior planet across the sky from the sun would not occur. In the Copernican system it must occur regardless of the particular rates at which the planets revolve in their orbits.]

But since all these [Spheres] have one center it is necessary that the space between the convex side of Venus's Sphere and the concave side of Mars's must also be viewed as a Sphere concentric with the others, capable of receiving the Earth with her satellite the Moon and whatever is contained within the Sphere of the Moon-for we must not separate the Moon from the Earth, the former being beyond all doubt nearest to the latter, especially as in that space we find suitable and ample room for the Moon.

We therefore assert that the center of the Earth, carrying the Moon's path, passes in a great circuit among the other planets in an annual revolution round the Sun; that near the Sun is the center of the Universe; and that whereas the Sun is at rest, any apparent motion of the Sun can be better explained by motion of the Earth. Yet so great is the Universe that though the distance of the Earth from the Sun is not insignificant compared with the size of any other planetary path, in accordance with the ratios of their sizes, it is insignificant compared with the distances of the Sphere of the Fixed Stars.

I think it easier to believe this than to confuse the issue by assuming a vast number of Spheres, which those who keep Earth at the center must do. We thus rather follow Nature, who producing nothing vain or superfluous often prefers to endow one cause with many effects. Though these views are difficult, contrary to expectation, and certainly unusual, yet in the sequel we shall, God willing, make them abundantly clear at least to mathematicians.

Given the above view - and there is none more reasonable - that the periodic times are proportional to the sizes of the Spheres, then the order of the Spheres, beginning from the most distant is as follows. Most distant of all is the Sphere of the Fixed Stars, containing all things, and being therefore itself immovable. It represents that to which the motion and position of all the other bodies must be referred . . . . Next is the planet Saturn, revolving in 30 years. Next comes Jupiter, moving in a 12 -year circuit; then Mars, who goes round in 2 years. The fourth place is held by the annual revolution [of the Sphere] in which the Earth is contained, together with the Sphere of the Moon as on an epicycle. Venus, whose period is 9 months, is in the fifth place, and sixth is Mercury, who goes round in the space of 80 days.

In the middle of all sits Sun enthroned. In this most beautiful temple could we place this luminary in any better position from which he can illuminate the whole at once? He is rightly called the Lamp, the Mind, the

Ruler of the Universe; Hermes Trismegistus names him the Visible God, Sophocles' Electra calls him the All-seeing. So the Sun sits as upon a royal throne ruling his children the planets which circle round him. The Earth has the Moon at her service. As Aristotle says, in his On [the Generation of] Animals, the Moon has the closest relationship with the Earth. Meanwhile the Earth conceives by the Sun, and becomes pregnant with an annual rebirth.

So we find underlying this ordination an admirable symmetry in the Universe, and a clear bond of harmony in the motion and magnitude of the Spheres such as can be discovered in no other wise. For here we may observe why the progression and retrogression appear greater for Jupiter than Saturn, and less than for Mars, but again greater for Venus than for Mercury [a glance at Figure 32 will show that the closer the orbit of a planet is to the orbit of the earth, the larger the apparent retrograde, motion of that planet must be - an additional harmony of Copernicus' system]; and why such oscillation appears more frequently in Saturn than in Jupiter, but less frequently in Mars and Venus than in Mercury [the earth will lap a slowly moving superior planet more frequently than it laps a rapid one, and conversely for an inferior planet]; moreover why Saturn, Jupiter and Mars are nearer to the Earth at opposition to the Sun than when they are lost in or emerge from the Sun's rays. Particularly Mars, when he shines all night [and is therefore in opposition], appears to rival Jupiter in magnitude, being only distinguishable by his ruddy color; otherwise he is scarce equal to a star of the second magnitude, and can be recognized only when his movements are carefully followed. All these phenomena proceed from the same cause, namely Earth's motion.

That there are no such phenomena for the fixed stars proves their immeasurable distance, because of which the outer sphere's [apparent] annual motion or its [parallactic] image is invisible to the eyes. For every visible object has a certain distance beyond which it can no more be seen, as is proved in optics. The twinkling of the stars, also, shows that there is still a vast distance between the farthest of the planets, Saturn, and the Sphere of the Fixed Stars [for if the stars were very near Saturn, they should shine as he does], and it is chiefly by this indication that they are distinguished from the planets. Further, there must necessarily be a great difference between moving and non-moving bodies. So great is this divine work of the Great and Noble Creator!

Throughout this crucially important tenth chapter Copernicus' emphasis is upon the "admirable symmetry" and the "clear bond of harmony in the motion and magnitude of the Spheres" that a suncentered geometry imparts to the appearances of the heavens. If the sun is the center, then an inferior planet cannot possibly appear far from the sun; if the sun is the center, then a superior planet must be
in opposition to the sun when it is closest to the earth; and so on and on. It is through arguments like these that Copernicus seeks to persuade his contemporaries of the validity of his new approach. Each argument cites an aspect of the appearances that can be explained by either the Ptolemaic or the Copernican system, and each then proceeds to point out how much more harmonious, coherent, and natural the Copernican explanation is. There are a great many such arguments. The sum of the evidence drawn from harmony is nothing if not impressive.

But it may well be nothing. "Harmony" seems a strange basis on which to argue for the earth's motion, particularly since the harmony is so obscured by the complex multitude of circles that make up the full Copernican system. Copernicus' arguments are not pragmatic. They appeal, if at all, not to the utilitarian sense of the practicing astronomer but to his aesthetic sense and to that alone. They had no appeal to laymen, who, even when they understood the arguments, were unwilling to substitute minor celestial harmonies for major terrestrial discord. They did not necessarily appeal to astronomers, for the harmonies to which Copernicus' arguments pointed did not enable the astronomer to perform his job better. New harmonies did not increase accuracy or simplicity. Therefore they could and did appeal primarily to that limited and perhaps irrational subgroup of mathematical astronomers whose Neoplatonic ear for mathematical harmonies could not be obstructed by page after page of complex mathematics leading finally to numerical predictions scarcely better than those they had known before. Fortunately, as we shall discover in the next chapter, there were a few such astronomers. Their work is also an essential ingredient of the Copernican Revolution.

## Revolution by Degrees

Because he was the first fully to develop an astronomical system based upon the motion of the earth, Copernicus is frequently called the first modern astronomer. But, as the text of the De Revolutionibus indicates, an equally persuasive case might be made for calling him the last great Ptolemaic astronomer. Ptolemaic astronomy meant far more than astronomy predicated on a stationary earth, and it is only with respect to the position and motion of the earth that Copernicus broke with the Ptolemaic tradition. The cosmological

